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Spin-stabilized solar sail for displaced solar orbits

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ABSTRACT

An optical force model is used to investigate the stability of a flat spinning solar sail in a displaced solar orbit. The solar sail can be stabilized in the orbit by design of the spinning rate and the sail structure. The orbital and attitude dynamics are studied separately. The orbit is stable as the sail attitude keeps fixed with respect to the sunlight, as does that of a perfectly reflecting solar sail. The attitude is stable as long as the spin angular velocity is much larger than the orbital angular velocity. The stability of the individual components cannot guarantee the stability of the entire system since the orbit and attitude interact with each other. Therefore, the coupled dynamics of the orbit and attitude are used to study the overall stability; the results show that the coupled system is also stable. It should be noted that the orbit and attitude are critically not asymptotically stable. The analysis only provides the necessary conditions for stability because a linearization is performed. To numerically verify the nonlinear stability of the true nonlinear system, the dynamical equations are simulated for a time that is longer than the mission life.

1. Introduction

Solar sails were initially proposed to take advantage of the solar radiation pressure to propel a spacecraft by means of a large membrane mirror. Later, it was found that a solar sail could enable new classes of missions that would be difficult for chemically propelled spacecraft, such as the Geostorm Warning Mission [20], Polar Observer [1,9] and the GeoSail [12,17]. The Polar Observer Mission concept involves a spacecraft following the Earth's rotation around the Sun in order to have a continuous, hemispherical view of one of the Earth's poles. Such a view will enhance the observation and telecommunications for the high latitude and polar region. Observation of the polar region currently relies on satellites in highly inclined low Earth orbits, which restricts them to observe only narrow swaths of the polar region during each passage. A pole-sitter remaining at a fixed position above a pole could provide a full, real time hemispherical view of the polar region. The pole-sitter utilizes a non-Keplerian orbit (NKO). The application of such an orbit to a pole-sitter mission was first proposed by Driver [1] and later by Forward [2], and an extensive two-body investigation has been performed by McInnes [13]. Continuous low-thrust propulsion, discrete impulses, or a solar sail are required to maintain the NKO [14]. The two-body problem permits analysis of displaced geostationary orbits using hybrid propulsion [8], while three-body applications include NKOs in the Earth–Moon system for lunar far-side communication [22] and lunar south pole coverage [7]. McKay summarized the methods of maintaining NKOs using both solar sailing and low-thrust propulsion in two-body and three-body models [18].

This paper examines the possibility of using a spin-stabilized solar sail for an NKO application. It is well known that the orbit of a solar sail can be controlled by adjusting its attitude. There are two main categories of stabilization for the attitude: three-axis stabilization and spin stabilization. A spinning solar sail usually requires no support structure, and can be deployed by the centrifugal force. However, it has disadvantages in attitude maneuvering since the large angular momentum due to the spin has to be changed. Previous studies primarily investigated the attitude control of the three-axis stabilized solar sail. Conventional methods of attitude control, such as thrusters and momentum wheels, are expensive for solar sails because they add complexity and mass to the system, and consume propellant. Therefore, devices altering the center of pressure and center of mass locations within the solar sail system to generate control torques are preferred, e.g. tip vanes [11], moving mass, or a mass on a gimbaled boom [23]. Rizvi et al. have studied a spinning heliogyro that uses the sail blade pitching motion to produce the control torques [21]. IKOROS uses the Reflectivity Control Device mounted on the edge of the sail membrane to generate a torque by changing the induced force on the surface of each small element [3]. Mimasu et al. utilize the solar radiation torque to change the direction of the spin axis by controlling its spin rate [19]. Kirpichnikov initially proposed that a solar sail could be made passively stable with respect to the sunlight vector by the design of its shape [10]. This passive control concept has been shown to always produce stability when used

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Nomenclature

μ	Gravitational constant of the Sun	$\mathrm{N\,m^2/kg}$
β	Lightness number of the solar sail, dimensionless	
m	Mass of the sailcraft	kg
$\boldsymbol{a}_s$	Solar radiation pressure acceleration	$\mathrm{m/s^2}$
P	Solar radiation pressure	Pa
A	Area of the solar sail	$\mathrm{m^2}$
$\boldsymbol{n}$	Unit vector along the sail normal, a unit vector	
$\boldsymbol{s}$	Unit vector along the Sun-sail direction, a unit vector	
$\boldsymbol{r}$	Position vector of the sail with respect to the Sun	m
ρ	Radius of the displaced orbit (distance from the sail to the Z axis)	m
z	Displaced height (distance from the sail to the ecliptic plane)	m
α	Pitch angle of the sail normal with respect to sunlight	rad
δ	Clock angle of the sail normal with respect to sunlight	rad
θ	Rotational angle along the sail normal	rad
$\mathbf{M}_s$	Solar radiation pressure torque	N m
A_{o2rf}	Transition matrix from the orbital frame to the rotation-free frame, an orthogonal matrix	
A_{o2i}	Transition matrix from the orbital frame to the inertial frame, an orthogonal matrix	
$\boldsymbol{\omega}_{rf2i}$	Angular velocity of the rotation-free frame	rad/s
$\boldsymbol{\omega}_{spin}$	Angular velocity of the body-fixed frame relative to the rotation-free frame	rad/s
$\boldsymbol{\omega}_{o2i}$	Angular velocity of the orbital frame	rad/s
$\boldsymbol{\Omega}$	Angular velocity of the body-fixed frame	rad/s
$\mathbf{H}$	Angular momentum of the sail	$\mathrm{kg\,m^2/s}$
$\mathbf{L}$	Vector from the center of mass to the center of pressure	m
ε_{xy}	Component of $\mathbf{L}$ in the plane perpendicular to the normal	m
ε_z	Component of $\mathbf{L}$ along the normal	m
$\mathbf{I}$ (I_x, I_y, I_z)	Moment of inertia	$\mathrm{kg\,m^2}$
$\tilde{\omega}$	$\tilde{\omega} = \sqrt{\mu/r^3}$	rad/s
$\mathbf{0}_{m\times n}$	A zero matrix of $m \times n$	
$\mathbf{E}_m$	An identity matrix of order m	

in NKOs [4,15]. However, a special shape is necessary to obtain the force balance condition in a displaced orbit for a perfectly reflecting model. Previous studies have examined the passive control design for shapes including pyramids [4] and cones [6], where the shape of a perfectly reflective sail is designed to make it passively stable on the NKO. For a perfectly reflective spinning solar sail, the shape has to be a cone to satisfy the force and torque balance conditions. However, an SRP (solar radiation pressure) torque exists for an optically flat sail, and the torque-induced precession can match the orbital motion exactly. Therefore, it is possible to design a flat spinning solar sail that is passively stable on the NKO.

This paper considers an optically flat sail that is spin-stabilized. First, the orbit dynamics of the solar sail are analyzed, and it is found that the orbit is stable if the attitude remains fixed with respect to the sunlight. Then, the attitude dynamics of a spinning solar sail in an NKO are studied, and the sail structure parameters are set to produce the attitude profile required by NKO. To be stable in an NKO, the sail has to precess to match the orbit angular velocity. The precession is induced by the SRP torque. Therefore, the spin rate is designed to balance the SRP toque and angular velocity of precession. As the design is accomplished, the orbit and attitude dynamics are used to analyze the orbit and attitude stability. Finally, more detailed simulations are conducted to verify the stability.

2. Solar sail model and definition of reference frames

A flat solar sail with an optical force model [16,24] is assumed. The propulsive sail acceleration can be written as

$$\boldsymbol{a}_S = \frac{PA}{m}(\boldsymbol{n}\cdot\boldsymbol{s})\left[b_1\boldsymbol{s} + (b_2\boldsymbol{n}\cdot\boldsymbol{s} + b_3)\boldsymbol{n}\right] \tag{1}$$

where b_1, b_2, and b_3 are the force coefficients related to the thermo-optical properties of the reflective film. The force coefficients are assumed to be independent of the sail attitude, and constant over the whole mission. The values of the coefficients are given in Table 1; the solar gravitational constant and astronomical unit are also given. These values are used in all of the succeeding simulations.

The expression for the sail acceleration, Eq. (1), can be more conveniently expressed with the introduction of the dimensionless lightness number, which is defined as

$$\beta = \frac{2PAr^2}{m\mu} \tag{2}$$

Then, the SRP acceleration can be written as

$$\boldsymbol{a}_S = \beta\frac{\mu}{2r^2}(\boldsymbol{n}\cdot\boldsymbol{s})\left[b_1\boldsymbol{s} + (b_2\boldsymbol{n}\cdot\boldsymbol{s} + b_3)\boldsymbol{n}\right] \tag{3}$$

To analyze the orbit and attitude dynamics of the solar sail, four reference frames are defined as follows:

Inertial frame ($OXYZ$): The origin is the mass center of the Sun, the Z axis is perpendicular to the ecliptic plane, the OXY plane lies in the ecliptic plane, and the X axis points to the equinox.

Orbital frame ($ox_{\mathrm{orb}}y_{\mathrm{orb}}z_{\mathrm{orb}}$): The origin is the mass center of the sail, the z_{orb} is along the Sun-sail line, x_{orb} is perpendicular to the plane formed by the Z and z_{orb} axes, and the y_{orb} axis forms a right-hand frame with the x_{orb} and z_{orb} axes.

Rotation-free frame ($ox_{rf}y_{rf}z_{rf}$): The origin is the mass center of the sail, the z_{rf} is along the sail normal, and the x_{rf} and y_{rf} axes are in the plane of sail film and are free of self-rotation of the sail.

Body-fixed frame ($ox_{\mathrm{body}}y_{\mathrm{body}}z_{\mathrm{body}}$): The origin is the mass center of the sail, the z_{body} axis is along the sail normal, and the x_{body} and y_{body} axes are in the plane of the sail film. The relative orientation of the body-fixed frame with respect to the rotation-free frame is described by a rotation along the sail normal (z_{rf} axis or z_{body} axis).

3. Orbit dynamics of a solar sail

A two-body model is used to analyze the orbit dynamics of the solar sail. The forces exerted on the solar sail are solar gravity and the SRP force. The cylindrical coordinates (ρ, θ, z) are used to describe a position vector $\boldsymbol{r}$ in the inertial frame. ρ is the distance from the Sun to the projection of $\boldsymbol{r}$ in the ecliptic plane, θ is the angle between the X axis and the projection of $\boldsymbol{r}$ in the ecliptic plane, and z is the distance from the solar sail to the ecliptic plane. The transition between the cylindrical coordinates and Cartesian coordinates is given by

$$\begin{cases} X = \rho\cos\theta \\ Y = \rho\sin\theta \\ Z = z \end{cases} \tag{4}$$