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Recursive estimation for descriptor systems with multiple packet dropouts and correlated noises ☆

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ABSTRACT

In this paper, the problem of recursive estimation is studied for a class of descriptor systems with multiple packet dropouts and correlated noises. The multiple packet dropouts phenomenon is considered to be random and described by a binary switching sequence that obeys a conditional probability distribution. The autocorrelated measurement noise is characterized by the covariances between different time instants. The descriptor system is transformed into a regular line system with an algebraic constraint. By using an innovation analysis method and the orthogonal projection theorem, recursive estimators including filter, predictor and smoother are developed for each subsystem and the process noise. Further, the recursive filter, predictor and smoother are obtained for the original descriptor system with possible multiple packet dropouts phenomenon and correlated noises. Simulation results are provided to demonstrate the effectiveness of the proposed approaches.

1. Introduction

In the past decades, the state estimation problem for descriptor systems has received significant attention, since these systems arise naturally in extensive practical application areas, such as economic system, robotics system, electric network system and chemical system. Different from non-descriptor systems, the future dynamics of descriptor systems can affect the present values of the state, and this leads to more difficulties in the research. So far, a great deal of state estimators have been available in literature [1–4,9,12,19,23,28]. For example, the problem of robust Kalman filtering for uncertain descriptor systems has been investigated in [12]. In [1,19], the estimation problem for descriptor systems has been transformed into the estimation problem for non-descriptor systems. The descriptor Kalman filter based on the least-square method has been studied in [2]. In [3,28], the optimal estimation for descriptor systems has been treated based on modern time-series analysis method. The distributed Kalman filter fusion for descriptor systems has been studied in [4,9], where an optimal fusion criterion weighted by block-diagonal matrices is used.

The problem of state estimation with correlated noises has attracted recurring interests in recent years [11,15–18], This is due to the fact that correlated noises are commonly encountered in engineering practice. In [18], the state estimation for discrete-time systems with cross-correlated noises has been treated based on an optimal weighted matrix sequence, where the process noises and measurement noises are cross-correlated. The optimal Kalman filtering fusion problem for systems with cross-correlated sensor noises has been dealt with in [11,16]. However, the estimators in [11,16,18] only deal with the correlated noises at the same time instant. Song et al. [17] presented a Kalman-type recursive filter for systems with finite-step autocorrelated process noises. The filtering problem with finite-step cross-correlated process noises and measurement noises has been investigated in [8].

On the other hand, with the development of network technologies, the networked control systems (NCSs) have received significant attention in many practical applications for its advantages of low cost, great mobility, simple installation and implementation. However, the cost of these advantages is the need for communication and handing the network-induced uncertainties. In the NCSs, the limited capacity communication networks that are generally shared by a group of systems have brought us new challenges in the analysis and design state estimators with time-delays or/and packet dropouts (also called missing measurements). Several results have been proposed when time-delays are deterministic, see [13] and references therein. However, time-delays and packet dropouts in networked systems are inherently random. Recently, the binary switching sequence has been employed to describe time-delays or/and multiple packet dropouts for its simplicity and practicality [7,10,14,20–22,24–27]. In [21,22], the problem of robust filtering for uncertain systems with missing measurements has been investigated. The least-mean-square filtering problem for one-step random sampling delay has been studied in

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[25,26]. Unfortunately, the filters designed in [25,26] are suboptimal since a colored noise due to augmentation has been treated as a white noise. The filtering problem for systems with random measurement delays and multiple packet dropouts has also been discussed in [14,20]. In [27], the optimal non-fragile filtering problem for dynamic systems with finite-step autocorrelated measurement noises and multiple packet dropouts has been studied. The problem of optimal filtering for uncertain systems with different delay rates sensor network and autocorrelated process noises has also been investigated in [7]. It should be noted that all the literature mentioned above has been concerned with non-descriptor systems, and the corresponding results for descriptor systems are relatively few. Recently, Feng and Zeng [6] presented recursive estimators for descriptor systems with different delay rates, where the delay of each sensor is restricted to at most one unit delay. However, in many applications, the delay of each sensor may not obey this restriction. Up to now, to the best of the authors' knowledge, the recursive estimation problem has not yet been addressed for descriptor systems with multiple packet dropouts and correlated noises, and this situation motivates our current study.

In this paper, we aim at solving the recursive estimation problem for a class of descriptor systems with multiple packet dropouts and correlated noises. The multiple packet dropouts phenomenon is described by a binary switching sequence satisfying a conditional probability distribution. The measurement noise is autocorrelated. Without loss of generality, the measurement noise is assumed to be one-step autocorrelated. The descriptor system is transformed into a regular linear system with an algebraic constraint. By using an innovation analysis approach and the orthogonal projection theorem (OPT), recursive estimators including filter, predictor and smoother are developed for each subsystem and the process noise. Further, we can obtain the recursive filter, predictor and smoother for the original descriptor system. *The main contribution of this paper is threefold*: 1) *to the best of the authors' knowledge, this is the first time that the multiple packet dropouts phenomenon is considered in descriptor systems*; 2) *the considered autocorrelated measurement noise which is characterized by the covariances between different time instant is intractable*; 3) *the recursive estimators including filter, predictor and smoother are developed for the proposed descriptor system*. A numerical simulation example is used to demonstrate the effectiveness of the proposed estimation schemes in this paper.

The remainder of the paper is organized as follows. In Section 2, the recursive estimation problem is formulated for a class of descriptor systems with multiple packet dropouts and correlated noises. The recursive estimators including filter, predictor and smoother are derived in Section 3. In Section 4, a simulation example is provided to illustrate the usefulness of the theory developed in this paper. We end the paper with some concluding remarks in Section 5.

Notation. The notation used in the paper is fairly standard. The superscripts "T" and "+" stand for matrix transposition and pseudo-inverse, respectively. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all real matrices of dimension $m\times n$, and I and 0 represent the identity matrix and zero matrix, respectively. The notation $P>0$ means that P is real symmetric and positive definite, and diag(...) stands for block-diagonal matrix. δ_{k-j} is the Kronecker delta function, which is equal to unity for $k=j$ and zero for $k\neq j$. In addition, $\mathcal{E}\{x\}$ means mathematical expectation of x and Prob$\{\cdot\}$ represents the occurrence probability of the event "$\cdot$". Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem formulation

Consider the following descriptor system with multiple packet dropouts and correlated noises:

$$E\vec{s}_{k+1}=F\vec{s}_k+G\vec{\omega}_k, \tag{1}$$

$$\vec{y}_k=C\vec{s}_k+\vec{v}_k, \tag{2}$$

$$y_k=\lambda_k\vec{y}_k+(1-\lambda_k)y_{k-1}, \tag{3}$$

where $\vec{s}_k\in\mathbb{R}^n$ is the state of the system to be estimated. $\vec{y}_k\in\mathbb{R}^m$ is the measured output of the sensor, $y_k\in\mathbb{R}^m$ is the measurement received by estimators. $\vec{\omega}_k\in\mathbb{R}^q$ is the process noise with zero mean and covariance $\vec{Q}_k>0$. E, F, G, and C are known matrices with appropriate dimensions. $\lambda_k\in\mathbb{R}$ is a binary switching white sequence and has the statistic properties as follows:

$$\begin{aligned}&\text{Prob}\{\lambda_k=1\}=\mathcal{E}\{\lambda_k\}=\beta_k,\\&\text{Prob}\{\lambda_k=0\}=1-\mathcal{E}\{\lambda_k\}=1-\beta_k,\\&\sigma_k^2=\mathcal{E}\{(\lambda_k-\beta_k)^2\}=(1-\beta_k)\beta_k,\end{aligned} \tag{4}$$

where $\beta_k\in[0\ 1]$ is a known real time-varying positive scalar and λ_k is assumed to be uncorrelated with other noise signals. $\vec{v}_k\in\mathbb{R}^m$ is the zero mean measurement noise correlated with $\vec{\omega}_k$ and has the statistic properties as follows:

$$\begin{aligned}&\mathcal{E}\{\vec{v}_k\vec{v}_t^T\}=\vec{R}_k\delta_{k-t}+\vec{R}_{k,k-1}\delta_{k-t-1}+\vec{R}_{k,k+1}\delta_{k-t+1},\\&\mathcal{E}\{\vec{\omega}_k\vec{v}_t^T\}=\vec{S}_k\delta_{k-t},\end{aligned} \tag{5}$$

where $\vec{R}_k=\vec{R}_k^T$. From (5), it is known that the measurement noise $\vec{v}_k$ is one-step autocorrelated. The measurement noise at time k is correlated with the measurement noises at time $k-1$ and $k+1$ with covariances $\vec{R}_{k,k-1}$ as well as $\vec{R}_{k,k+1}$, respectively.

Remark 1. The measurement model (3) was first introduced in [14] and has been employed in [20] to describe the multiple packet dropouts phenomenon. In measurement (3), if $\lambda_k=\lambda_{k-1}=0$ while $\lambda_{k-2}=1$, i.e. the measurements at time k and $k-1$ are lost and the measurement at time $k-2$ will be used at $k-1$ and k. Thus, the measurement (3) can be used to describe multiple packet dropouts phenomenon.

Assumption 1. *E is singular, i.e.* det $E=0$. rank $E=n_1$, $n_1<n$.

Using Assumption 1 and the singular value decomposition [5], there exist two nonsingular matrices U and V such that

$$UEV=\begin{bmatrix}\Delta & 0\\ 0 & 0\end{bmatrix},\qquad \Delta=\text{diag}(\mu_1,\mu_2,\ldots,\mu_{n_1}),\ \mu_i>0,\ i=1,2,\ldots,n_1. \tag{6}$$

By defining

$$UFV=\begin{bmatrix}F_{11} & F_{12}\\ F_{21} & F_{22}\end{bmatrix},\qquad UG=\begin{bmatrix}G_1\\ G_2\end{bmatrix},\qquad CV=[C_1\ C_2],\qquad \vec{s}_k=V\begin{bmatrix}s_{1,k}\\ s_{2,k}\end{bmatrix}, \tag{7}$$

the system (1)–(3) can be rewritten as follows:

$$s_{1,k+1}=\Delta^{-1}F_{11}s_{1,k}+\Delta^{-1}F_{12}s_{2,k}+\Delta^{-1}G_1\vec{\omega}_k, \tag{8}$$

$$0=F_{21}s_{1,k}+F_{22}s_{2,k}+G_2\vec{\omega}_k, \tag{9}$$

$$\vec{y}_k=C_1s_{1,k}+C_2s_{2,k}+\vec{v}_k, \tag{10}$$

$$y_k=\lambda_k\vec{y}_k+(1-\lambda_k)y_{k-1}, \tag{11}$$

where $s_{1,k}\in\mathbb{R}^{n_1}$ and $s_{2,k}\in\mathbb{R}^{n_2}$, $n_2=n-n_1$.