Aerospace Science and Technology 32 (2014) 212–222

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Analysis of optimal aircraft cruise with fixed arrival time including wind effects

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ARTICLE INFO

Article history:
Received 12 April 2013
Received in revised form 10 October 2013
Accepted 15 October 2013
Available online 21 October 2013

Keywords:
Minimum-fuel cruise
Fixed arrival time
Singular optimal control
Wind effects

ABSTRACT

Minimum-fuel cruise at constant altitude with the constraint of a fixed arrival time is analyzed, including the effects of average horizontal winds. The analysis is made using the theory of singular optimal control. The optimal control is of the bang-singular-bang type, and the optimal trajectories are formed by a singular arc and two minimum/maximum-thrust arcs joining the singular arc with the given initial and final points. The effects of average horizontal winds on the optimal results are analyzed, both qualitatively and quantitatively. The influence of the initial aircraft weight and the given cruise altitude is analyzed as well. Two applications are studied: first, the cost of meeting the given arrival time under mismodeled winds, and, second, the cost of flight delays imposed on a nominal optimal path. The optimal results are used to assess the optimality of cruising at constant speed; the results show that the standard constant-Mach cruise is very close to optimal. Results are presented for a model of a Boeing 767-300ER.

1. Introduction

An important problem in air traffic management (ATM) is the design of aircraft trajectories that meet certain arrival-time constraints at given waypoints, for instance at the top of descent, at the initial approach fix, or at the runway threshold (estimated time of arrival). The final-time constraint may be defined, for example, by a flight delay imposed on the nominal (preferred) trajectory. These are four-dimensional (4D) trajectories, which are a key element in the trajectory-based-operations (TBO) concept proposed by SESAR and NextGen for the future ATM system (for example, Bilimoria and Lee [5] analyze aircraft conflict resolution with an arrival time constraint at a downstream waypoint). Also important in ATM is the design of optimal flight procedures that lead to energy-efficient flights. In practice, the airlines consider a cost index (CI) and define the direct operating cost (DOC) as the combined cost of fuel consumed and flight time, weighted by the CI; their goal is to minimize the DOC. When the flight time is fixed, the objective is to minimize fuel consumption.

In the analysis of aircraft trajectories with fixed flight time, wind effects are of primary importance, because changes in wind speed modify the flight time (over a given range), and therefore lead to changes in the speed profiles required to keep the final-time constraint.

Minimum-DOC trajectories have been studied by different authors. For example, Barman and Erzberger [1] and Erzberger and Lee [11] analyze the minimum-DOC problem for global trajectories (climb-cruise-descent), considering steady cruise and taking the aircraft mass as constant; wind effects are considered in Ref. [1] in the case of short-haul missions (range below 500 km). Burrows [7] also analyzes the minimum-DOC problem for global trajectories, without the assumption of constant mass, but with the assumption that the cruise segment takes place in the stratosphere. The particular case of minimum-fuel cruise (CI equal to zero) has been considered by others. For example, Schultz and Zagalsky [25], Speyer [27], Schultz [24], Speyer [28] and Menon [20] analyze the optimality of the steady-state cruise, taking the aircraft mass as constant; wind effects are not considered.

Fuel-optimal trajectories with fixed arrival time are studied by Sorensen and Waters [26], Burrows [8], Chakravarty [10] and Williams [30], who analyze the 4D fuel-optimization problem as a minimum-DOC problem with free final time, that is, the problem is to find the time cost for which the corresponding free-final-time, DOC-optimal trajectory arrives at the assigned time. The effects of horizontal winds are considered in Refs. [10,30]: in Ref. [10] the cost of absorbing delays is analyzed, in a scenario formed by the final cruise (400 nmi) and the descent, for altitude-dependent winds, and in Ref. [30] the effects of mismodeled winds in a scenario formed by the final cruise (200 nmi) and the descent segments are studied, for the case of constant winds; wind effects on the whole cruise segment, however, are not considered.

In this paper, an analysis of minimum-fuel cruise with fixed arrival time, at constant altitude, in the presence of horizontal winds, is presented. The analysis is made using the theory of singular optimal control (see Bell and Jacobson [2]). The problem is unsteady, with variable aircraft mass. The initial and final speeds are

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1270-9638/$ – see front matter
http://dx.doi.org/10.1016/j.ast.2013.10.005

Nomenclature

a	speed of sound	t_f	flight time
c	specific fuel consumption	T	thrust
D	drag	T_M	maximum thrust
g	gravity acceleration	V	aerodynamic speed
h	altitude	w	wind speed
H	Hamiltonian	W	aircraft weight
K	constant value of the Hamiltonian	x	horizontal distance
L	lift	x_f	range
m	aircraft mass	λ	adjoint variable
m_F	fuel mass	π	thrust control parameter
M	Mach number	Δt_f	flight delay
S	switching function	Δw	mismodeled wind
t	time	Ω	singular-arc parameter

given, so that the structure chosen for the optimal control is bang-singular-bang, with the optimal paths formed by a singular arc and two minimum/maximum-thrust arcs joining the singular arc with the given initial and final points. The singular arc in the case of no winds is studied in Franco et al. [13].

Singular optimal control theory has been used, among other works, to analyze maximum-range cruise at constant altitude (Pargett and Ardema [21], Rivas and Valenzuela [22]), minimum-cost cruise including both the DOC and the arrival-error cost associated to not meeting the scheduled time of arrival (Franco and Rivas [12]), and maximum-range unpowered descents in the presence of altitude-dependent winds (Franco et al. [14]).

The main objective of this work is to present a quantitative analysis of the effects of average horizontal winds on the optimal trajectories and control laws that lead to minimum fuel consumption while meeting the final-time constraint. The influence of the initial aircraft weight and the given cruise altitude on the optimal results is also analyzed. From the operational point of view, two applications are studied: first, the fuel penalties associated to mismodeled winds are estimated, that is, the cost of meeting the given time of arrival under mismodeled winds is quantified; and, second, the cost of flight delays imposed on a nominal optimal path is quantified as well.

The optimal trajectories define speed laws in which the Mach number varies along the singular arc. These optimal solutions, which are a reference for optimal performance, are used to assess the optimality of the standard constant-Mach cruise procedure commonly used in practice (according to air traffic regulations). The comparison with optimal results shows that the performance of the constant-Mach cruise is very close to optimal.

Results are presented for a model of a Boeing 767-300ER (a typical twin-engine, wide-body, long-range transport aircraft), with realistic aerodynamic and propulsive models, and for constant winds, which represent average winds along the cruise.

The outline of the paper is as follows: the problem is formulated in Section 2, the numerical procedure used to solve the optimal problem is described in Section 3, the constant-Mach cruise procedure is described in Section 4, the results are presented in Section 5, and some conclusions are drawn in Section 6; the aircraft model is described in Appendix A.

2. Problem formulation

As already indicated, because the flight time is fixed, the objective is to minimize the fuel consumption for a given range, that is, to minimize the following performance index

$$J = \int_0^{t_f} cT \, dt \tag{1}$$

with t_f fixed, subject to the following constraints

$$\begin{aligned} \dot V &= \frac{1}{m}(T - D) \\ \dot m &= -cT \\ \dot x &= V + w \end{aligned} \tag{2}$$

which are the equations of motion for cruise at constant altitude and constant heading, in the presence of a horizontal wind (see Jackson et al. [15]). In these equations, the drag is a general known function $D(V, m)$, which takes into account the remaining equation of motion $L = mg$; the thrust $T(V)$ is given by $T = \pi T_M(V)$, where π models the throttle setting, $0 < \pi_{min} \leqslant \pi \leqslant \pi_{max} = 1$, and $T_M(V)$ is a known function; the specific fuel consumption, $c(V)$, is also a known function; and the wind speed $w(h)$ is a known function which depends on the given altitude h. Thus, in this problem there are three states, speed (V), aircraft mass (m) and distance (x), and one control, π. The initial values of speed, aircraft mass and distance (V_i, m_i, x_i), and the final values of speed and distance (V_f, x_f) are given. The final value of aircraft mass (m_f) is unspecified. (Note that D, T_M and c also depend on the given altitude h.)

The Hamiltonian of this problem is given by

$$H = c\pi T_M + \frac{\lambda_V}{m}(\pi T_M - D) - \lambda_m c\pi T_M + \lambda_x(V + w) \tag{3}$$

where λ_V, λ_m and λ_x are the adjoint variables. Note that H is linear in the control variable, so that it can be written as $H = \overline{H} + S\pi$, where $\overline{H}$ and the *switching* function S are given by

$$\begin{aligned} \overline{H} &= -\lambda_V \frac{D}{m} + \lambda_x(V + w) \\ S &= \left[\frac{\lambda_V}{m} - (\lambda_m - 1)c\right] T_M \end{aligned} \tag{4}$$

The necessary conditions for optimality are summarized next (see Ross [23]):

1) The equations defining the adjoints:

$$\begin{aligned} \dot\lambda_V &= -\frac{\partial H}{\partial V} = -\lambda_x + \frac{\lambda_V}{m}\frac{\partial D}{\partial V} - \left[\frac{\lambda_V}{m} - (\lambda_m - 1)c\right]\pi \frac{dT_M}{dV} \\ &\quad + (\lambda_m - 1)\frac{dc}{dV}\pi T_M \\ \dot\lambda_m &= -\frac{\partial H}{\partial m} = \frac{\lambda_V}{m}\left[\frac{\pi T_M - D}{m} + \frac{\partial D}{\partial m}\right] \end{aligned}$$