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On three-dimensional global linear instability analysis of flows with standard aerodynamics codes

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ABSTRACT

The development of a general Jacobian-free approach for the solution of large-scale global linear instability analysis eigenvalue problems by coupling a time-stepping algorithm with industry-standard second-order accurate aerodynamic codes is presented. The three-dimensional lid-driven cavity, a challenging flow in the context of required computational resources and physical complexity, has been chosen for validation. Results in excellent agreement with the literature have been obtained by using the proposed theoretical methodology coupled with the incompressible solver of the open-source toolbox OpenFOAM. The moderate computational resources required for the solution of the TriGlobal eigenvalue problem using this method opens up a new avenue for the performance of instability analysis of flows of engineering relevance.

1. Introduction

Global linear instability analysis theory [76] plays an essential role in the investigation of the sequence of physical mechanisms leading laminar flow in complex, spatially inhomogeneous geometries through transition to turbulence. The theory deals with the temporal and spatial evolution (growth/decay) of small-amplitude perturbations superimposed upon a steady or unsteady laminar base flow. The assumption of asymptotic (long-time) instability leads to a generalized large-scale eigenvalue problem, the challenging numerical solution of which provides the spectrum of linear global modes composed of the modal frequencies and amplification/damping rates. Such numerical solution can be obtained within a matrix-forming or a matrix-free/Jacobian-free framework. The main difference between the two approaches is that matrix-forming strategies provide access to larger subsets of the full spectrum at the cost of large computational memory (RAM memory) while matrix-free methods provides smaller subsets of the full spectrum at the cost of long time integration (CPU time). Both frameworks make use of subspace projection-iterative methods such as the Arnoldi iteration, based on the Krylov-subspaces [67,7], which is one of the most effective techniques to solve the resulting generalized eigenproblem when formation of the full discretized matrix is impractical due to the problem size. The Arnoldi method delivers a window of the eigenspectrum but favors the eigenvalues with the largest modulus, thus a transformation of the spectrum is required in order to introduce an eigenvalue shift towards the interesting part of the spectrum. The shift-invert transformation was first introduced in fluid mechanics in a matrix-forming context by Natarajan and Acrivos [57], while the time-stepping exponential transformation was first developed by Erikson and Rizzi [28] in a Jacobian-free framework. A recent review [76] provides a discussion of the suite of matrix transformation methods used up to the time of writing that article, while recent progress and challenges using these two frameworks has been recently presented by Gómez et al. [34].

Although a large number of studies using the two different frameworks have reported significant insight in instability mechanisms over the last four decades in relatively complex flows with one homogeneous spatial direction, such as attachment lines [62] or open cavities [19], most flows of practical engineering significance still remain unexplored. The principal reason arises from the difficulties associated to the analysis of turbulent flows, an issue not discussed here; the interested reader is referred to the works of Biau et al. [13] and Nichols and Lele [58] amongst others. The second reason for the relatively little attention paid to the analysis of flows of industrial interest is that basic state of most practical flows are three-dimensional depending in an inhomogeneous manner on the three spatial directions, and no assumptions regarding spatial homogeneity can be made; the related analysis context is known as TriGlobal linear stability. Although the single parameter of the instability problem in this situation in incompressible flow is the Reynolds number, the cost of performing a complete

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Nomenclature

x, y, z	Spatial coordinates	$\mathbf{A}$	Jacobian matrix
$\bar{\mathbf{u}}$	Base flow components	$\mathbf{H}$	Hessenberg matrix
$\mathbf{u}'$	Perturbation components	ϵ	Perturbation magnitude
Re	Reynolds number	ϵ_m	Numerical tolerance
λ	Eigenvalue in the stability analysis	ϵ_o	Initial order of perturbation magnitude
$\hat{\mathbf{u}}$	Eigenvector in the stability analysis	$\varepsilon(t)$	Integration residual at time t
$\mathbf{K}_m$	Krylov subspace	Δt	Time step in temporal integration
m	Krylov subspace dimension	CFL	Courant number
τ	Integration time	N	Number of nodes in one spatial direction

parametric instability analysis can be prohibitively expensive when the matrix discretizing the eigenvalue problem is solved in a dense matrix-forming framework, as inferred from the work of Rodriguez and Theofilis [66], in which a $O(1)$ Tb RAM memory matrix was formed, stored and inverted for the solution of a BiGlobal problem.

Despite the fact that a high-order sparse matrix-forming method has recently been shown to provide $O(10^4)$ speed-up with respect to dense matrix-forming approaches [60], matrix-free methods remain the method of choice for TriGlobal linear instability analysis problem; a case study was provided by Gómez et al. [36,35]. The key advantage of matrix-free time-marching methods, over explicit formation of the Jacobian matrix is that the large-sized matrices describing spatial discretization of global linear instability analysis applications in both two or even more so in three inhomogeneous spatial directions resolved in a coupled manner is never formed. This enables the study of global linear stability problems on small-main-memory machines at the expense of long-time integrations. A rather complete discussion of time-stepping approaches for global linear instability has recently been presented by Barkley, Blackburn and Sherwin [11]. The first successful time-stepping methodology by Erikson and Rizzi [28] introduced a numerical differentiation of the DNS used with a temporal polynomial approximation. In that work, finite differences were used in order to study an inviscid incompressible flow over a NACA airfoil. Chiba [22] improved the Erikson and Rizzi approach by introducing a temporal exponential transformation using a full Navier–Stokes equations solver. Following Chiba's method, Tezuka and Suzuki [71,72] successfully solved the first TriGlobal problem ever. In parallel, Edwards et al. [27] developed a time-stepping methodology in conjunction with the linearized Navier–Stokes equations, which has been successfully used and popularized by Barkley et al. [10], Tuckerman et al. [79] and many others. Although these previously mentioned algorithms are able to provide only a part of the spectrum, recent matrix-free algorithms can provide access to the full spectrum using time-stepping approaches, e.g. Bagheri et al. [8] and elsewhere [54,61].

Despite these new capabilities for global stability analysis that recent sparse matrix-forming and matrix-free algorithms offer, only a number of canonical configurations with three inhomogeneous spatial directions have been analyzed with respect to their linear instability; to the best of the knowledge of the authors, these are the above-mentioned spheroid [72], an incompressible jet in a cross flow [9], sphere [59], and the three-dimensional, lateral-wall-bounded lid-driven cavity [32,53,30,35]. This lack of TriGlobal analyses in the literature can be attributed to the fact that the time-stepping matrix-free methodology requires a three-dimensional Direct Numerical Simulation (DNS) solver and the development and validation of a three-dimensional DNS capable of handling different geometries is non-trivial.

The goal of the present work is to present an algorithm for TriGlobal modal linear instability analysis that can overcome the excessive computational requirements of the matrix-forming techniques and the necessity of developing a three-dimensional direct numerical simulation solver for the specific task. This is accomplished by linking matrix-free/Jacobian-free instability algorithms with existing general purpose aerodynamic codes, the latter run in direct numerical simulation mode. Moreover, the necessity of flexibility and ability to handle complex geometries makes second-order standard aerodynamic codes the first candidate to be examined regarding their suitability for TriGlobal instability analysis. Although no work is known to date that deals with the numerical solution of large-scale TriGlobal eigenvalue problems using standard aerodynamic codes, leaving most problems of practical engineering significance still unexplored, second-order methods have been already successfully used in global linear instability theory both for the solution of the BiGlobal [25,56,43,4,5] and that of the TriGlobal linear EVP [32]. High-order accurate spectral element methods [45,15,77] or even finite elements [37,39] may provide a better convergence rate for a given resolution than second-order finite volumes methods while maintaining geometry flexibility, however the open-source code OpenFOAM code based on second-order finite-volume spatial discretization has been chosen for this work because of its flexibility, ease of performing source-code modifications and the ability this code offers to study different flow regimes in future research.

The three-dimensional, lateral-wall-bounded lid-driven cavity was chosen as a demonstration problem for TriGlobal linear instability analysis, since it permits examining two different aspects: physical complexity and computational efficiency. In addition, the non-unity aspect ratio configuration of this cavity flow, though well-studied from an experimental and a three-dimensional DNS point of view, has never been addressed as regards its TriGlobal linear instability.

From the point of view of physical complexity, the accurate description of the fully three-dimensional lid-driven cavity flow still remains inconclusive in many aspects, as stated in the recent work by Feldman & Gelfgat [30], although the analysis of the two-dimensional counterpart of the lid-driven cavity flow has become a benchmark problem in fluid mechanics and has been extensively reviewed [70,29,18].

The main reason for this lack of understanding is that the three-dimensional lid-driven cavity flow, as found experimentally by Koseff and Street [49,50] and numerically by Iwatsu et al. [42,41], presents a far more complicated structure that cannot be directly compared to the corresponding two-dimensional flow. The most important three-dimensional flow features are the Taylor–Görtler-Like (TGL) vortices [49,69] and corner eddies or end-wall vortices (EWV) [20,63] in the flow field. Fig. 1 shows schematically the geometry and the rest of flow features: primary eddy (PE), downstream secondary eddy (DSE), upstream secondary eddy (USE) and upstream upper eddy (UUE). Aidun et al. [1] performed lid-driven cavity experimental visualizations, presenting an excellent qualitative description of the state diagram of TGL structures and demonstrated the existence of different branches of n-cell