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Controllability analysis and attitude path planning of underactuated spacecraft systems

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ABSTRACT

The controllability analysis and attitude path planning are addressed for an underactuated spacecraft using two flywheels as actuators. Considering the spacecraft and flywheels as a whole system, we describe the dynamics of the system on an angular momentum level set such that the system is controllable with arbitrary initial momentum and direction of the torque singular vector. Moreover, an optimal performance index is proposed with the influence of friction torques in flywheels considered. With this index being optimized, Gauss Pseudospectral Method (GPM) is used to design the attitude path of the system, which satisfies the spacecraft maneuver requirement. Finally, simulation results show the effectiveness of the attitude path planning method.

1. Introduction

In aerospace applications, flywheels are widely used as actuators in small spacecrafts for their small masses, high precision and no consumption of propellant [14]. Since one flywheel can only produce control torque in a single direction, flywheels are used in arrays consisting of more than three flywheels to obtain torques along three independent directions, as well as redundancy. In long-time operation, a common problem is the failure of flywheels onboard a spacecraft. A possible result is that only two-dimensional torques are available for attitude control. In this case, the actuators are equivalent to two flywheels with their rotation axes installed along any two independent directions, and the system consisting of the spacecraft and flywheels is referred to as 'underactuated' [11].

The problem of attitude control of an underactuated spacecraft has been intensely investigated. Crouch first discussed the controllability of an underactuated spacecraft controlled with either jets or flywheels, and concluded that the system with less than three momentum wheels is never controllable [6]. Byrnes proved that such a system is impossible to be stabilized via smooth state feedback since the Brockett's necessary condition is violated [5]. M.H. Nadjim proposed a control algorithm for a spacecraft with two flywheels [15]; X. Ge solved the control of the underactuated spacecraft by using attitude motion planning, which is proved effective [8].

In the above analysis, it is noted that the controllability analysis of the system is based on the dynamics and kinematics of the underactuated spacecraft, and the controllers are designed based on the assumptions that the total angular momentum of the underactuated spacecraft is zero, and the normal vector of the control torque plane is along the principal inertia axis of the whole system. Indeed, since the flywheels can only produce two-dimensional torques, without these two assumptions, the system is never controllable on the underlying state space where the quaternion and angular rates are taken as state variables. However, we found that the system is controllable on a smaller level set, that is 'momentum level set', the introduction of which greatly promoted the usage of flywheels as actuators on spacecraft. It follows that these assumptions described a special case of momentum level set.

Since the strict assumptions limit the application of flywheels as actuators, a practical problem, never addressed as far as we know, is how to control an underactuated spacecraft using two flywheels with arbitrary initial momentum and arbitrary normal vector of the control torque plane. In this paper, the controllability analysis of a spacecraft with two flywheels is conducted on the angular momentum level sets, and the attitude path planning is designed.

In Section 2, mathematical model is constructed with the spacecraft and flywheels considered as a whole system. Differently from the mathematical description proposed by Crouch [6], the dynamics of the whole system is described on an angular momentum level set, on which the controllability of the system is discussed. Based on the conclusion of the controllability analysis, in Section 3, an optimal performance index is proposed for the system, and GPM is used to design the attitude path of the system. In

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Section 4, simulation of the system is conducted. We conclude the paper with Section 5.

2. Controllability analysis of the underactuated spacecraft

In R^4, we define the set $\aleph$ as a sphere with radius 1. Obviously, $\aleph$ is compact. The attitude kinematics of the spacecraft is described as follows [10]:

$$\dot{\boldsymbol{q}} = \xi(\boldsymbol{q})\boldsymbol{\omega} = \frac{1}{2}\begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix}\boldsymbol{\omega} \tag{1}$$

where $\boldsymbol{q} = [q_1\ q_2\ q_3\ q_4]^{\mathrm{T}} \in \aleph$ is the quaternion of the spacecraft attitude, which satisfies the following identity:

$$q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$$

$\boldsymbol{\omega} = [\omega_1\ \omega_2\ \omega_3]^{\mathrm{T}}$ is the angular velocity vector. The rotational dynamics of the rigid spacecraft with n flywheels is given by [12,22]:

$$\boldsymbol{I}\dot{\boldsymbol{\omega}} = \boldsymbol{\omega} \times \boldsymbol{B}\boldsymbol{H}_0 + \sum_{i=1}^{n} \boldsymbol{b}_i j_i \ddot{\theta}_i \tag{2}$$

where $\boldsymbol{I}$ is the inertia matrix, $\boldsymbol{B}$ is the transition matrix of the system, depending on the quaternion, $\boldsymbol{b}_i$ is the unite vector along the rotation axis of the ith flywheel in body coordinate; j_i, $\ddot{\theta}_i$ are the momentum inertia and the angular acceleration of the ith flywheel about the axis $\boldsymbol{b}_i$ respectively. $\boldsymbol{H}_0$ is the initial angular momentum of the whole system, i.e.

$$\boldsymbol{B}\boldsymbol{H}_0 = \boldsymbol{I}\boldsymbol{\omega} + \sum_{i=1}^{n} \boldsymbol{b}_i j_i \dot{\theta}_i \tag{3}$$

which can be derived via the law of angular momentum conservation. We define the matrix $\boldsymbol{A}$ as follows

$$\boldsymbol{A} = [\boldsymbol{b}_1, \ldots, \boldsymbol{b}_i, \ldots, \boldsymbol{b}_n]$$

The spacecraft described by Eqs. (1) and (2) is obviously controllable on the set $M = \aleph \times R^3$, if the rank r_0 of the matrix $\boldsymbol{A}$ equals 3.

In order to analyze the controllability of the system consisting of the spacecraft and two flywheels, the following two lemmas are given as the main tools [3,4,20,21].

Lemma 1 *(Poincaré). Let $(\aleph, \Omega)$ be a manifold with a volume form Ω and n_Ω its Borel measure. Let $\boldsymbol{f}_a$ be a time-independent, complete vector field such that its flow ϕ_t, $t \in R$, preserves the volume. Suppose A is a measurable subset of $\aleph$ with $0 < n_\Omega(A) < \infty$, which is also invariant under the flow of $\boldsymbol{f}_a$. Then for each measurable subset B of A with $n_\Omega(B) > 0$ and for any $T > 0$, there exists a $t > T$ such that $\phi_t(B) \cap B \neq \emptyset$.*

Lemma 2. *For the system*

$$\dot{x} = \boldsymbol{f}(x) + \sum_{i=1}^{m} \boldsymbol{g}_i(x) u_i, \qquad \boldsymbol{u} = [u_1, \ldots, u_m]^{\mathrm{T}} \in R^m$$

suppose that $\boldsymbol{f}$ is a Weak Positive Poisson Stability (WPPS) vector field. The system is controllable if $\boldsymbol{F} = \{\boldsymbol{f}, \boldsymbol{g}\}$ satisfies the Lie Algebra Rank Condition (LARC).

For the underactuated spacecraft using two flywheels, we denote the two independent rotation axes of the flywheels by $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$. In such a case, the rank r_0 of the matrix $\boldsymbol{A}$ equals 2. The flywheels cannot produce any momentum vector change or torque along the direction of $\boldsymbol{v} = \frac{\boldsymbol{b}_1 \times \boldsymbol{b}_2}{|\boldsymbol{b}_1 \times \boldsymbol{b}_2|}$. Such a unit vector $\boldsymbol{v}$ is called the singular direction, which satisfies the following equation:

$$\boldsymbol{v}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{H}_0 = \boldsymbol{v}^{\mathrm{T}}\boldsymbol{I}\boldsymbol{\omega} + \boldsymbol{v}^{\mathrm{T}}\sum_{i=1}^{2} \boldsymbol{b}_i j_i \dot{\theta}_i \tag{4}$$

where $\boldsymbol{v}^{\mathrm{T}}\sum_{i=1}^{2} \boldsymbol{b}_i j_i \dot{\theta}_i$ is constant. Eq. (4) is a physical constraint. First we consider a special case of operation: assume the initial angular momentum is zero, i.e. $\boldsymbol{H}_0 = 0$, and the unit vector $\boldsymbol{v}$ is along the principal inertia axis of the whole system. In this case, Eqs. (1) and (2) satisfy the LARC. In addition, the drift vector of Eqs. (1) and (2) is Poisson stable [6]. Using Lemma 2, we know that the underactuated spacecraft is controllable on the set M.

Remark 1. The two assumptions, together with Eq. (4) guaranteed $\omega_i \equiv 0$, $i = \{1 \text{ or } 2 \text{ or } 3\}$, which can be removed from the states variables of the system. So the dimensions of Eqs. (1) and (2) are reduced. The state equations describing the system on the set $M = \aleph \times R^2$ satisfy the LARC.

For the flywheels onboard a spacecraft, the friction due to low angular rates will introduce an obvious influence on the precision of output torques. The preset initial angular velocities with certain magnitudes will exclude this situation. Furthermore, the singular direction $\boldsymbol{v}$ deviates from any principal inertia axis of the whole system due to the existence of the inertia products and the arrangement of the flywheels. For these reasons, the system fails to meet the assumptions mentioned above, it follows that system does not satisfy the LARC, and is uncontrollable on the set M.

To investigate the controllability of the system, the dynamics of the system is described on an angular momentum level set. We define $H_0 \in R^3$ as the initial angular momentum of the system; $\dot{\boldsymbol{\theta}} = [\dot{\theta}_1\ \dot{\theta}_2] \in R^2$ as the angular velocity of the flywheels, $N_{H_0} \subseteq \aleph \times R^3 \times R^2$ as the corresponding angular momentum level set. According to Eq. (3), the map

$$\alpha : \aleph \times R^2 \to \aleph \times R^3 \times R^2$$

given by

$$f_{\boldsymbol{H}_0} = \left(\boldsymbol{q}, \boldsymbol{I}^{-1}\left(\boldsymbol{B}\boldsymbol{H}_0 - \sum_{i=1}^{2} \boldsymbol{b}_i j_i \dot{\theta}_i\right), \dot{\boldsymbol{\theta}}\right)$$

is a diffeomorphism between $\aleph \times R^2$ and $N_{\boldsymbol{H}_0}$. The kinematics of the spacecraft is rewritten as:

$$\dot{\boldsymbol{q}} = \xi(\boldsymbol{q})\boldsymbol{I}^{-1}\left(\boldsymbol{B}\boldsymbol{H}_0 - \sum_{i=1}^{2} \boldsymbol{b}_i j_i \dot{\theta}_i\right) \tag{5}$$

The state equation of the underactuated spacecraft with two flywheels is rewritten as:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}\boldsymbol{u} \tag{6}$$

where $\boldsymbol{x} = [\boldsymbol{q} \quad \dot{\boldsymbol{\theta}}]^{\mathrm{T}}$, and the control input $\boldsymbol{u} = [u_1 \quad u_2]^{\mathrm{T}} \in R^2$ is the angular accelerations of the flywheels. Eq. (6) is an affine nonlinear system, with the drift vector field $\boldsymbol{f}(\boldsymbol{x})$ being:

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} f_\alpha(\boldsymbol{x}) \\ [0]_{2\times 1} \end{bmatrix} \tag{7}$$

where [0] is the null matrix with an appropriate dimension, $\boldsymbol{f}_\alpha(\boldsymbol{x}) = \xi(\boldsymbol{q})\boldsymbol{I}^{-1}(\boldsymbol{B}\boldsymbol{H}_0 - \sum_{i=1}^{2} \boldsymbol{b}_i j_i \dot{\theta}_i)$. The control vector fields $\boldsymbol{g}_1$