Aerospace Science and Technology 27 (2013) 32–43

Contents lists available at SciVerse ScienceDirect

Aerospace Science and Technology

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Multiple target tracking based on homogeneous symmetric transformation of measurements

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ARTICLE INFO

Article history:
Received 2 September 2011
Received in revised form 28 April 2012
Accepted 11 June 2012
Available online 26 June 2012

Keywords:
Multiple target tracking
Symmetric measurement equation
Kalman filter

ABSTRACT

Symmetrical measurement equation, generated from homogeneous symmetric functions, has been proposed in this paper for tracking multiple targets. The observability condition, resultant measurement noise and its covariance for any number of particles arising from proposed symmetric transformation of measurement have been derived. The derived expression of resultant noise covariance is verified using Monte Carlo run. As a case study, three particles in motion are considered where positions and velocities of the particles are estimated using extended Kalman filter. From the simulation results it is found that the targets' identity is lost during estimation. The target tracks have been labeled by minimizing the sum of square errors over the permutation of states. The performance of estimator in terms of root mean square error is compared with the two types of symmetric transformation of measurements, namely sum of power and sum of product form, existing in literature. Results are also compared with optimal state estimator which assumes that the correct association between measurements and targets is known. From simulation it is observed that RMSEs of position and velocity are small in homogeneous form compared to those obtained from sum of power and product form.

1. Introduction

The research interest for simultaneous tracking of multiple objects is increasing rapidly as it finds many applications in surveillance [3], robotics [5], collision avoidance [6], econometrics [7] and signal processing just to name a few. The core problem is to track multiple targets in clutter environment where targets may originate or terminate at any instant of time and association between targets and measurements is unknown. Classical approach is to compute the association probabilities [17,18] between measurements and targets before estimation. The main drawback of such approach is its computational inefficiency as the complexity increases exponentially (or factorial) with the number of targets.

In an alternative approach [9–11,13,14], the computational complexity described above can be circumvented if the number of targets to be tracked at any instant is assumed to be known. The assumption may be justified with the availability of sensor (for example Radar) which is capable of collecting the information which can be used to infer the number of targets within the area of coverage. The key idea is to convert the measurement data with unknown association to a symmetrical measurement equation to estimate the states of the targets. In this way it is possible to estimate targets' state without even considering association between targets and measurements. Sometimes this type of filter is called as symmetrical measurement equation (SME) filter [11].

In this paper, a new type of symmetrical measurement transformation based on homogeneous symmetric function has been introduced to transform the measurement data, obtained from sensor to form symmetric measurement equation. The proposed form will be a new addition in the family of symmetrical measurement equations which consists of two types of measurement equations namely sum of power and sum of product form. The observability condition for the developed symmetric transformation of measurements has been derived in the form of a proposition. The expressions of resultant measurement noise and its covariance have been derived and the later has been verified using Monte Carlo run. The approach has been illustrated through a simple case study where motion of three particles is considered. It has been observed that although the targets' states have been estimated, the estimator fails to label the track of particles. To label the track, all the permutations of states have been considered and the estimated values of states are frozen for that permutation which has least sum of square error. A comparison of estimation accuracy among different types of symmetrical measurements and also

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1270-9638/$ – see front matter
http://dx.doi.org/10.1016/j.ast.2012.06.004

with the associated filter [10,11] (estimator with known correct association) has been made in terms of root mean square error (RMSE). Simulation results reveal that RMSEs of position and velocity are less in homogeneous form compared to that of obtained from sum of power and sum of product form.

It may not be irrelevant to mention here that there is no optimal or the "best" Gaussian filter available in literature for multiple target tracking with nonlinear measurements. The claim of polynomial filter introduced by Luca et al. [15] as optimal estimation approach [2] in target tracking is misleading. In the polynomial filter, first two moments are computed for certain types of polynomial nonlinearity using full Taylor series expansion thus the non-Gaussian pdf arises due to nonlinear process and measurement equations is approximated as Gaussian. The previous study reveals that [14] the performance of SME approach depends on the combination of symmetrical measurement equation and nonlinear estimator rather than on either individually. In this paper a systematic approach has been taken where the performance of different types of symmetrical measurement equation has been compared with the same nonlinear estimator. Obviously the study of different types of symmetrical equation with several other nonlinear filters remains under the scope of future work.

The paper is organized as follows: Section 2 presents the formulation of target tracking problem of N particles. Section 3 is focused on the development of new symmetric measurement equation generated from homogeneous symmetric functions characterized with its resultant measurement noise and its covariance. A case study of three particles in motion is considered as a simulation problem and results are discussed in Section 4. Concluding remarks are in Section 5.

2. Problem formulation

2.1. Process model

Let us consider N particles maneuvering in a three dimensional space. Being interested in estimating position and velocity of the particles, state vector is considered to be constituted with position and velocity of all the particles along three axes. So for N targets moving in space, state vector can be assumed as $X_k = [x_{1k}\ x_{2k}\ \ldots\ x_{NK}\ \ v_{1k}\ v_{2k}\ \ldots\ v_{Nk}]^T$, where x_{ik} and v_{ik} represent the positions and velocities of ith target at time kT, with T as sampling time and $i = 1, 2, \ldots, N$. For two particles moving along straight line, the state vector would be $X_k = [x_{1k}\ x_{2k}\ v_{1k}\ v_{2k}]^T$. If three particles move in 1D space the state vector would be $X_k = [x_{1k}\ x_{2k}\ x_{3k}\ v_{1k}\ v_{2k}\ v_{3k}]^T$. If two particles move in 2D space, there are horizontal and vertical components of position and velocity. So the state vector would be $X_k = [x_{x,1k}\ x_{y,1k}\ x_{x,2k}\ x_{y,2k}\ v_{x,1k}\ v_{y,1k}\ v_{x,2k}\ v_{y,2k}]^T$, where $x_{x,ik}$ and $v_{x,ik}$ are the x directional position and velocity of ith particle at any instant k. Similar expression of state vector can be obtained for N particles moving in three dimensional space. Depending upon the nature of maneuver, the positions and velocities of the targets are modeled in state space using the nonlinear difference equations

$$X_{k+1} = \gamma(X_k) + Bw_k \tag{1}$$

where $\gamma(.)$ is a nonlinear function and w_k is zero-mean Gaussian white noise with Q_k covariance.

2.2. Measurement model

Now suppose the sensor which provides noisy measurement of the position is located at the origin of the coordinate system. If we assume the sensor outputs are the individual positions of the targets with correctly known association between targets and measurements, the measurement equation becomes linear and can be written as:

$$Y_k = HX_k + u_k \tag{2}$$

Here $Y_k = [y_{1k}\ y_{2k}\ \ldots\ y_{Nk}]^T$ where y_{ik} is the ith sensor measurement data at time kT; H is measurement matrix and $u_k = [u_{1k}\ u_{2k}\ \ldots\ u_{Nk}]^T$ is the measurement noise. For example, if two particles move in 1D space the measurement data would be $Y_k = [y_{1k}\ y_{2k}]^T$. For three particles in 1D space the measurement vector is $Y_k = [y_{1k}\ y_{2k}\ y_{3k}]^T$. Similarly for two particles moving in a plane, the measurement data would be $Y_k = [y_{x,1k}\ y_{y,1k}\ y_{x,2k}\ y_{y,2k}]^T$. Similar expression can be written for any number of particles moving in 3D of space. As stated earlier, considering only targets' position as measurements, measurement matrix becomes $H = [I_{DN}\ 0_N]$ where D is the dimension of the space where the particles are moving and I_{DN} is the DN dimensional unity matrix. We also assume the measurement noise or sensor noise, u_k, is white Gaussian with zero mean and σ_k^2 covariance ($u_k \sim N(0, \sigma_k^2)$). As in this case both the process and measurement equations are linear, the problem can be solved using Kalman filter (KF) [1]. Since the estimator knows the correct association, this filter may also be called as associated filter [10,11].

Now let us consider the scenario where correct data association, i.e. the correct correspondence between the sensor measurements and respective target's position is not known. To circumvent data association problem, the sensor data are transformed through symmetrical transformation to form symmetrical measurement equation which is used by the filter to estimate position and velocity of the targets. In this respect three types of symmetrical transformation have been considered here. Among them, the two forms, sum of power [10] and sum of product [11] have appeared in literature. The homogeneous symmetry has been proposed in this paper.

2.2.1. Sum of power symmetry

The symmetric measurement equation for sum of power form for N particles can be written as

$$Y_k = \left[\sum_{i=1}^{N} y_{ik} \quad \sum_{i=1}^{N} y_{ik}^2 \quad \cdots \quad \sum_{i=1}^{N} y_{ik}^N\right]^T \tag{3}$$

where $y_{ik} = x_{ik} + u_{ik}$ is the measured position of ith particle at time instant kT in presence of noise u_{ik} which is assumed to be white Gaussian with zero mean and σ_k^2 covariance.