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Improving variable-fidelity surrogate modeling via gradient-enhanced kriging and a generalized hybrid bridge function

Zhong-Hua Han [a,*,1], Stefan Görtz [b,2], Ralf Zimmermann [b,2]

[a] National Key Laboratory of Science and Technology on Aerodynamic Design and Research, Department of Fluid Mechanics, School of Aeronautics, Northwestern Polytechnical University, 710072 Xi'an, PR China

[b] German Aerospace Center (DLR), D-38108 Braunschweig, Germany

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ABSTRACT

Variable-fidelity surrogate modeling offers an efficient way to generate aerodynamic data for aero-loads prediction based on a set of CFD methods with varying degree of fidelity and computational expense. In this paper, direct Gradient-Enhanced Kriging (GEK) and a newly developed Generalized Hybrid Bridge Function (GHBF) have been combined in order to improve the efficiency and accuracy of the existing Variable-Fidelity Modeling (VFM) approach. The new algorithms and features are demonstrated and evaluated for analytical functions and are subsequently used to construct a global surrogate model for the aerodynamic coefficients and drag polar of an RAE 2822 airfoil. It is shown that the gradient-enhanced GHBF proposed in this paper is very promising and can be used to significantly improve the efficiency, accuracy and robustness of VFM in the context of aero-loads prediction.

1. Introduction

From an aerodynamic point of view, an aircraft is defined by comprehensive datasets regarding performance, loads and handling characteristics. This data, which needs to be determined at a given timescale and cost for every possible flight condition and aircraft configuration, is used to design the structure of the aircraft and the flight control system. Currently, this data is obtained mainly from costly wind tunnel tests or by using hand-book methods. The use of higher-fidelity and thus more accurate but also more time consuming CFD methods has been, up to now, impossible due to the large number of load cases that need to be evaluated to achieve aircraft certification. Only a subset of the required data can be computed with high-fidelity CFD at present. The "brute-force" approach of computing all relevant data with high-fidelity CFD is currently not feasible and methods for reducing the computational costs are sought after.

The long-term goal of the work described herein is the development of a process chain for the efficient numerical prediction of all certification-relevant aerodynamic data for the elastic aircraft over the full flight envelope, based on a hierarchy of CFD methods from low to high fidelity. The idea is to use Variable-Fidelity Modeling (VFM) to produce this data in a reduced time-frame with guaranteed accuracy and minimum computational effort.

The VFM method [1,2] for aero-loads prediction uses a set of CFD methods with varying degrees of fidelity and computational expense (potential theory, Euler equations, and RANS equations) or a single physical model evaluated on meshes of varying refinement to approximate the unknown aerodynamic data as a function of input parameters such as Mach number, angle of attack, etc., while reducing the number of expensive high-fidelity computations. VFM as discussed in this paper has the same meaning as "multi-fidelity modeling", "variable-complexity modeling", "data fusion" or "data merging".

The most popular method currently used for VFM is a correction-based method. The correction is called "bridge function", "scaling function" or "calibration". The correction can be multiplicative, additive or hybrid multiplicative/additive. Multiplicative bridge function was used for variable-fidelity optimization [3] since 1993. It is used to "locally" scale the low-fidelity function to approximate the high-fidelity function. When it is combined with the trust region method [4–6], the optimization cost can be greatly reduced. Additive bridge function was then developed as an "global" correction and has become the most popular method for variable-fidelity optimization [7–9] or for data fusion [10]. It was proven to be more accurate and robust than a multiplicative bridge function. To represent more complicated correlation between low- and high-fidelity functions, a hybrid multiplicative/additive bridge function method was proposed in [11–13] for variable-fidelity optimization. A VFM framework for aero-loads prediction was developed by the authors as described in Refs. [1,2]. It was verified

* Corresponding author.
E-mail addresses: hanzh@nwpu.edu.cn (Z.-H. Han), Stefan.Goertz@dlr.de (S. Görtz), Ralf.Zimmermann@dlr.de (R. Zimmermann).

1 Research Scientist, former affiliation: German Aerospace Center (DLR), Institute of Aerodynamics and Flow Technology, $C^2A^2S^2E$.

2 Research Scientist, Institute of Aerodynamics and Flow Technology, $C^2A^2S^2E$.

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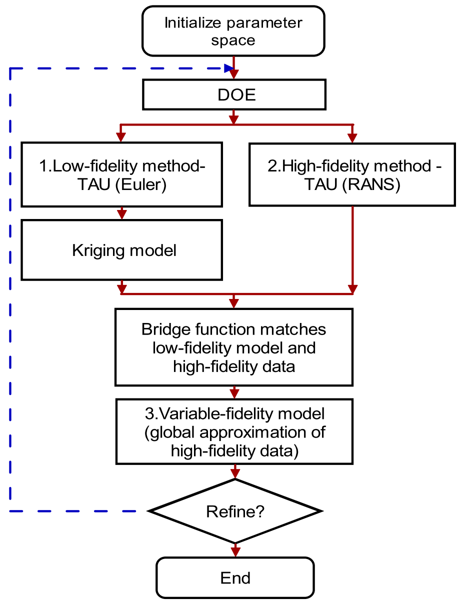

Fig. 1. VFM framework for aero-loads prediction.

for analytic problems and preliminarily demonstrated and evaluated for the construction of global approximation models of the aerodynamic coefficients and drag polar of a 2D airfoil. It was demonstrated that the approximation method, the bridge function and the refinement strategy are the key elements for constructing a VFM model. Although the VFM method has been shown to be promising, more research has to be carried out to develop new methods and technologies that will further improve its efficiency, accuracy and robustness.

Aware of the importance of improving the available VFM algorithm in the context of aero-loads prediction, this paper focuses on Gradient-Enhanced Kriging (GEK) in combination with a new Generalized Hybrid Bridge Function (GHBF) methodology. The new methods and technologies are verified for analytic problems and demonstrated by constructing a global approximation model for the aerodynamic coefficients and the drag polar of an RAE 2822 airfoil.

The paper is organized as follows: Section 2 gives a brief overview of the framework of the VFM method we are concerned with. Section 3 deals with the details of the GEK method, including the GEK predictor and its Mean Squared Error (MSE), the choice of the correlation models, model fitting and remarks on the difference between GEK and cokriging. Section 4 is about the development of the GHBF for VFM. Finally, Section 5 shows some numerical examples.

2. Variable-fidelity modeling framework

This article focuses on the development of a VFM method that is especially well suited for predicting and modeling the aerodynamic data of aircraft throughout their full flight envelope. VFM is essentially a model management methodology. It uses a set of CFD methods with varying degrees of fidelity and computational expense or a single physical model evaluated on meshes of varying resolutions. A low-fidelity CFD method is used to automatically compute hundreds or thousands of solutions at points in the parameter space selected with a Design of Experiments (DoE) tool. The remainder of the parameter space is “filled in” using interpolation procedures. A few points in the parameter space are computed using a high-fidelity CFD method. These points are also selected using a DoE tool. Then, data fusion is used to combine the data stemming from the different methods, with low-fidelity data indicating trends and a small number of high-fidelity data correcting the absolute values. This is done by using so-called bridge functions. The model is adaptively refined by inserting additional samples based on various refinement criteria. The VFM framework was designed for constructing a model that can approximate the high-fidelity data throughout the parameter space. A flow chart is depicted in Fig. 1. The basic steps of this framework are as follows:

- *Step 1 – Initialization*: Define the unknown aerodynamic function (integrated or distributed) to be modeled; specify the parameter space by defining the independent variables and their range.
- *Step 2 – Sampling*: Two sets of sample points (called samples in the following) are generated based on Design-of-Experiment (DoE) theory; one set with a large number of samples to be evaluated with the low-fidelity method and one significantly smaller set for evaluation with the high-fidelity method.
- *Step 3 – Sample point evaluation*: The aerodynamic data at the samples are calculated with the low- and high-fidelity CFD methods, respectively.
- *Step 4 – Low-fidelity surrogate model and bridge function*: Based on the sampled low-fidelity data, a kriging model is constructed as a surrogate model to the low-fidelity CFD method (called low-fidelity kriging model). Based on the difference between the low-fidelity surrogate model and the high-fidelity data at the locations of the high-fidelity samples, a kriging-based bridge function is constructed to match the low- and high-fidelity CFD methods.
- *Step 5 – Initial VFM construction*: The low-fidelity surrogate model is corrected with the bridge function and an initial VFM is constructed.
- *Step 6 – Refinement*: iterative refinement is performed by adding additional samples until a criterion for termination is fulfilled.
- *Step 7 – Final VFM for aero-loads prediction*: Based on the final VFM, the parameter space can be probed in “real-time” for aerodynamic data at any point in the parameter space within the limits prescribed in Step 1 or a database of aerodynamic data can be efficiently generated by filling in the remainder of the parameter space using the VFM.

3. Gradient-enhanced kriging

Kriging is a statistical interpolation method suggested by Krige [14] in 1951 and mathematically formulated by Matheron [15] in 1963. Kriging estimation depends on the spatial correlations between given sample points to be interpolated. Gradient-Enhanced Kriging (GEK) denotes the extension of kriging to models where the gradient information is incorporated into the construction of the kriging model to improve the accuracy of the prediction for a given number of samples. In turn, the efficiency of constructing an approximation model for an unknown aerodynamic function can be improved as fewer samples are needed for a given level of accuracy. There are two ways to incorporate the gradient information at samples, which lead to two different methods: direct GEK and indirect GEK (see Refs. [16,17]).

In the case of direct GEK [20–22], the gradient information is directly included in the kriging equation system by adding the weighted sum of the gradients to the weighted sum of the data. The additional weights are calculated by changing the kriging equation system to include the correlation between the data and the gradients. These correlations are modeled by differentiating the correlation function. A formal mathematical derivation of the kriging equation system and construction of the correlation function as well as correlation vector are necessary. In contrast to indirect GEK, where a finite difference step size has to be determined for