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# Sun–Earth–Moon autonomous orbit determination for quasi-periodic orbit about the translunar libration point and its observability analysis

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ABSTRACT

In this paper, the Sun–Earth–Moon (i.e., SEM) autonomous navigation problem is investigated for the Lissajous orbit and the quasi-halo orbit about the translunar libration point. Generally, the SEM navigation method can offer a convergent estimation by using orientation information. However, due to the unstable nature of the translunar libration orbit, it is still indispensable to further prove that only orientation information can guarantee the convergence. Therefore, three sensor configurations are studied to find an appropriate sensor configuration for translunar libration probe. In order to achieve this goal, a new navigation dynamic model with standard ephemerides is proposed. The observability analysis is used to evaluate the feasibility of each sensor configuration. Autonomous navigation is obtained by extended Kalman filtering. Simulations show that the sensor configuration of using orientation information from the spacecraft to the Sun, the Earth and the Moon and one Doppler measurement can satisfy both the economical efficiency and reliability.



## 1. Introduction

The five libration points were first found by Euler and Lagrange in the Restricted Three-Body Problem (i.e., R3BP) [5]. In the last decades, periodic orbits in the three-body regime have successfully served as the basis for trajectory design in various missions [8], for instance, International Sun–Earth Explorer-3 (i.e., ISEE-3). However, less attention was paid to autonomous orbit determination technique, which is interested in this paper.

All navigation information used to determine the orbits of libration point probes is generated by Earth-based tracking sensors. For instance, (i) ISEE-3 was the first libration point mission to be tracked by S-band radiometric data on an irregular basis, in which, a batch processor was used for the orbit determination and the accuracy of the orbit determination was about 6 km [12]. (ii) Solar Heliospheric Observatory (i.e., SOHO) was mainly tracked with the 26-m Deep Space Network (i.e., DSN) antennas, with some 34-m and 70-m antennas data as well [3]. The orbit determination accuracy for SOHO was 7 km. (iii) The navigation for the Advanced Composition Explorer (i.e., ACE) was three hours of 26-m DSN tracking data every day, with some additional tracking data from the 34-m antennas whose orbit determination error was estimated to be about 10 km [27]. (iv) The Microwave Anisotropy Probe (MAP) was tracked for at least 45 minutes every day from the 34-m or 70-m DSN antennas and its orbit determination accuracy reached 2 km [17]. Dunham and Farquhar [4] listed eight future libration point missions, which, if successful, will increase the need for DSN tracking. Beckman [1] suggested that an autonomous navigation system for libration point orbits could eliminate the need for DSN. Besides, it has been well established that Earth-tracking navigation can only provide medium accuracy for station-keeping, which calls for more accurate navigation technique.

Several different methods of autonomous orbit determination with different measurement types [2,11,21–25] for interplanetary missions or Earth-centered missions have been proposed. However, few references about the autonomous navigation for libration point could be found until 2005. Hill and Born proposed a new method called Linked, autonomous, interplanetary satellite orbit navigation (i.e., LiAISON Navigation), which used scalar Satellite-to-Satellite Tracking observations to estimate the orbits of all the participating spacecrafts simultaneously [13–15]. It was an example to demonstrate the feasibility of the autonomous navigation. However, there were two limitations need to be pointed out. First, the LiAISON Navigation is a kind of constellation autonomy, which often corresponds to more cost, complexity and less reliability. Moreover, usually, it is harder to coordinate the launch of multiple spacecraft, not to mention the risk that a loss of one spacecraft will lead to the failure of whole mission in the constellation.

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Last, the dynamics used in LiAISON Navigation is the bicircular four-body model which makes several assumptions about the real Earth–Moon–Sun system. As well known, potential truncation error will be induced.

In order to overcome the limitations of previous studies, this paper proposes an "individual" autonomous orbit determination technique for the Earth–Moon libration probe and simulates this orbit determination technique with a more accurate model. Based on the special location of the Earth–Moon L2 quasi-periodic orbits (i.e., EML2) including quasi-halo orbits and Lissajous orbits, the explicitly regular motions and the outstanding optical properties of the Sun, the Earth and the Moon, SEM could be a better choice than other autonomous navigation schemes. It is a method that makes use of the information provided by the Sun, the Moon and the Earth, such as the orientation information and the radial velocity relative to the Moon or the Earth. The SEM navigation method has been applied successfully into the Earth-centered strongly-stable orbits (e.g., Microcosm Autonomous Navigation System) [16,18]. Since translunar libration orbits are unstable, there is still a lot of work needed to be done to confirm whether the SEM method can be utilized in EML2, such as: (I) Generally, the SEM method can offer a convergent estimation by using three different orientation information sensors during Earth-centered mission [19]. Yet further verification is still indispensable to prove that orientation information can satisfy the convergence for EML2. (II) For Earth-centered strongly-stable orbits, navigation can provide convergent results by collecting measurement information for several periods of the orbit. However, for EML2, initial errors could trigger a fast divergence of the unstable state and drift a probe far away from the nominal orbit. The navigation system should provide convergent results within a short period and the results need to be accurate enough for the station-keeping system.

The contribution of this paper is twofold: (I) A dynamic model for translunar libration probe with full consideration of both solar influence and the eccentricity of the Moon's orbit is proposed, in which the standard ephemerides (e.g., DE405) are used to express the states of the Sun and the Moon. (II) Three sensor configurations for SEM are investigated on both quasi-halo orbit and Lissajous orbit.

The rest of this paper is organized as follows. In Section 2, the derivation of a precise navigation dynamic model with standard ephemerides is presented as well as the SEM autonomous navigation methods. The observability analysis is introduced in Section 3 to compare the observability for three sensor configurations. Simulations of all the cases are given in Section 4. Conclusions are given in Section 5.

## 2. Navigation system modeling

### 2.1. Navigation dynamics

In order to describe the motion of a probe accurately, we develop a more accurate dynamic model with the standard ephemerides, which takes the eccentricity of the Moon's orbit and Sun's gravitational influence into full consideration. Compared to existing results on this venue, the proposed navigation model has fewer assumptions about the real Earth–Moon–Sun system than the bicircular four-body model and restricted three-body model [13,14].

Before proceeding further, the definition of the coordinate systems is introduced in Fig. 1.

**J2000:** The J2000 Geocentric Equatorial Coordinate System (J2000 $O$-$\boldsymbol{XYZ}$): Origin at the center of the Earth; $X$-axis points the vernal equinox at noon on January 1, 2000; $Z$-axis points to the North Pole at this time; $Y$-axis completes the right-handed coordinate system.

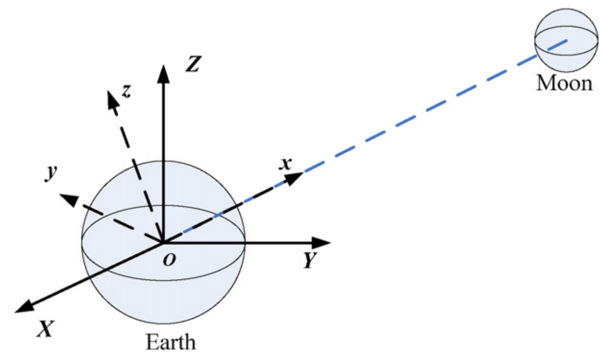


**Fig. 1.** The J2000 coordinate system and the Geocentric rotating coordinate system.

**GRC:** Geocentric Rotating Coordinate System (GRC $O$-$\boldsymbol{xyz}$): Origin at the center of the Earth; $x$-axis points to the center of the Moon; $z$-axis points to the instantaneous direction of the Moon's orbital angular momentum; $y$-axis completes the right-handed coordinate system.

The coordinate system $O$-$\boldsymbol{XYZ}$ is considered as an inertial coordinate system throughout this paper, so the absolute angular velocity of the rotating coordinate system $O$-$\boldsymbol{xyz}$ is the angular velocity relative to the J2000, which is denoted as $\boldsymbol{\omega}$.

If the influence of the Earth's non-spherical gravitation and the disturbances of other planets are neglected, a translunar libration probe's navigation dynamic model in the J2000 coordinate system is

$$\ddot{\boldsymbol{r}}_P = -\frac{\mu_E}{r_p^3}\boldsymbol{r}_P + \mu_M\left(\frac{\boldsymbol{r}_M-\boldsymbol{r}_P}{r_{PM}^3} - \frac{\boldsymbol{r}_M}{r_M^3}\right) + \mu_S\left(\frac{\boldsymbol{r}_S-\boldsymbol{r}_P}{r_{PS}^3} - \frac{\boldsymbol{r}_S}{r_S^3}\right) \tag{1}$$

where $\boldsymbol{r}_P$ denotes the position vector from the center of mass of the Earth to the translunar libration probe in J2000, $\boldsymbol{r}_S$ and $\boldsymbol{r}_M$ denote the Sun's position vector and the Moon's position vector relative to the center of the Earth in the J2000; and $r_{PM} = \|\boldsymbol{r}_M - \boldsymbol{r}_P\|$, $r_{PS} = \|\boldsymbol{r}_S - \boldsymbol{r}_P\|$; $\mu_E$, $\mu_M$, $\mu_S$ are the gravitational constants for the Earth, the Moon and the Sun, respectively.

Eq. (1) is a dynamic model based on Newton's classical mechanics. In order to make Eq. (1) fit for navigation system without adding more assumptions, we define that $\boldsymbol{r}_P^b$ is the position vector from the center of mass of the Earth to the translunar libration probe in the GRC. $\dot{\boldsymbol{r}}_P^b$ and $\ddot{\boldsymbol{r}}_P^b$ are the relative speed and the relative acceleration in the GRC. Then we can obtain Eq. (2):

$$\ddot{\boldsymbol{r}}_P = \ddot{\boldsymbol{r}}_P^b + 2\boldsymbol{\omega}\times\dot{\boldsymbol{r}}_P^b + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}_P) + \dot{\boldsymbol{\omega}}\times\boldsymbol{r}_P \tag{2}$$

The result from the combination of Eq. (1) and Eq. (2) is as follows:

$$\ddot{\boldsymbol{r}}_P^b = -2\boldsymbol{\omega}\times\dot{\boldsymbol{r}}_P^b - \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}_P) - \dot{\boldsymbol{\omega}}\times\boldsymbol{r}_P - \frac{\mu_E}{r_p^3}\boldsymbol{r}_P + \mu_M\left(\frac{\boldsymbol{r}_M-\boldsymbol{r}_P}{r_{PM}^3} - \frac{\boldsymbol{r}_M}{r_M^3}\right) + \mu_S\left(\frac{\boldsymbol{r}_S-\boldsymbol{r}_P}{r_{PS}^3} - \frac{\boldsymbol{r}_S}{r_S^3}\right) \tag{3}$$

The last term in Eq. (3) results from the direct solar perturbation. The effects of the indirect solar perturbation and the Moon's orbital eccentricity are contained in the expression of $\boldsymbol{\omega}$.

According to [26], the angular velocity of the GRC relative to the J2000 can be written with the ephemerides in the GRC as Eq. (4):

$$\boldsymbol{\omega} = \begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \begin{bmatrix}\frac{r_M}{h_M^2}\left(\frac{\mu_S}{r_{MS}^3} - \frac{\mu_S}{r_S}\right)(\boldsymbol{r}_S\cdot\boldsymbol{h}_M)\\ 0\\ \frac{h_M}{r_M^2}\end{bmatrix} \tag{4}$$