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Inverse identification of continuously distributed loads using strain data

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ABSTRACT

Operational load and stress data are useful for structural integrity management and damage prognosis of aerospace systems. Identifying aerodynamic loads by monitoring strain is not easy because the loads are distributed continuously over the structure's surface. In this study, we propose a flexible method for interpolating a continuous load distribution in order to identify the full-field aerodynamic load from strain data acquired at a number of discrete points. Our method uses the conventional finite element method and pseudo-inverse matrix, and we further extend it by coupling with an aerodynamical equation. Numerical simulations show that this extension improves the estimation accuracy when only a limited amount of strain data is available. The effects of measurement error are also discussed. It is concluded that the rank reduction method improves the estimation accuracy and that use of a proper aerodynamical restriction can suppress the adverse effect of measurement error.

1. Introduction

Structural Damage Prognosis (DP) is a promising technology that is expected to be applied to aerospace systems in near future. DP is defined as the estimation of a system's remaining useful life based on behavioral prediction models that combine information from usage monitoring, structural health monitoring (SHM), past, current and anticipated future environmental and operational conditions, original design assumptions, component and system level tests, and maintenance [7]. In discussing DP, Farrar et al. distinguish usage monitoring, the process of acquiring operational load data from a structure or a system, from health monitoring, the process of identifying the presence of damage and quantifying its extent [6].

Günther has reviewed loads and usage monitoring programs including new integrated systems [9]. Molent and Aktepe [15] and Aktepe and Molent [1] have reviewed the Australian Individual Aircraft Tracking (IAT) programs and concluded that IAT has been beneficial in comparing operational usage with design one.

Although usage and load monitoring have a long history in the field of aerospace, they continue to attract attention due to advances in sensors and data processing technologies. Current monitoring tools include flight hour/flight/landing cycle counting, and fatigue metering based on load factor, strain gauges, etc. [15]. These tools determine the levels of overall flight loads, such as wing root bending moment and/or torque [16].

These monitoring systems and programs do not measure pressure distributions or aerodynamic loads, although a better understanding of such loads would be useful and could improve fatigue monitoring capabilities [4]. The objective of the present study is to develop a method to identify continuously distributed (aerodynamic) loads from strain data measured at discrete locations.

The determination of loads from measured structural responses (strains) is an inverse problem. Mathematically, inverse problems are ill-posed and their solutions do not necessarily satisfy conditions of existence, uniqueness and stability. Therefore, a special approach such as regularization is required to obtain an accurate and stable solution.

Maniatty et al. investigated solutions for inverse elastic problems based on the finite element method with a regularization procedure to impose smoothness on the solution [14]. Maniatty also discussed the regularization procedure coupled with a statistical approach [13].

Schnur and Zabaras studied spatial regularization and recommended first-order regularization over zero- and second-order, but noted that the selection of the regularization constants is difficult [17]. They also introduced a polynomial approximation solution technique called the keynode method which provides a stable solution without regularization. Shkarayev et al. developed a finite element based methodology involving an inverse formulation that uses measured surface strains to recover applied loads [18]. They employed a parametric approximation where regularization was not required to obtain a stable solution. Coates et al. extended Shkarayev's method by approximating any function by its respective Fourier Cosine series and utilizing strain data to determine the coefficients [5]. Kirby and co-workers presented shape recovery

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Nomenclature

e_r	error factor	$[s]$ or s_{ji}	$m_s \times m_p$ matrix to transform p_i to ε_j^m
ERMS	error root mean square	$[s^+]$ or s_{ji}^+	generalized inverse matrix of $[s]$
m_p	number of nodes for pressure	x, y, z	Cartesian coordinates
m_s	number of strain data	β	penalty number
p_i $(i = 1, \ldots, m_p)$	nodal estimated pressure	ε_i^m $(i = 1, \ldots, m_s)$	measured strain data

for a beam based on strain data using a polynomial representation [12].

A learning algorithm for a multi-layer neural network was investigated by Cao et al. to determine the relationship between load and strain [3,4]. This method can avoid the mathematical difficulties inherent in inverse analysis but the mechanical ingredient of the obtained network is not clear.

Recent developments in optical strain sensors provided another motivation for this study. Fiber optic strain sensors (FOSS) have many advantages over conventional strain gauges such as strength, light weight, and immunity to electromagnetic interference and moisture. One of the authors developed a long gauge FBG (Fiber Bragg Grating) strain sensor using the OFDR (Optical Frequency Domain Reflectometry) technique. OFDR has the exciting potential to measure strains at arbitrary positions along a fiber [10,11]. Such recent developments indicate that in the near future, strain data measurements will be obtainable from thousands of points with little effort. This will enable practical load identification techniques based on inverse analysis, which requires a large amount of data to give an accurate and stable solution.

Tessler and co-workers developed a novel inverse finite element method (iFEM) to reproduce the full-field structural displacements from measured surface strains in plate and shell structures [19,20]. A combination of FOSS and iFEM was shown to reconstruct the full-field structural deformations of a cantilevered beam subjected to an imposed near-tip deflection at the free end [21].

In the present study we propose a flexible representation of continuous load distribution, and identify the full-field aerodynamic load from strain data measured at a number of discrete points using the conventional finite element method and a pseudo-inverse matrix. A unique feature of our method is the use of a mesh of triangular elements to approximately interpolate the continuous pressure distribution. This mesh is independent of the finite element model of the structure and thus the degree of freedom of the inverse analysis and/or the locations of the nodes can flexibly be controlled to obtain a stable solution. We further extend the method by coupling with an aerodynamical equation. Numerical simulations show that this extension improves the estimation accuracy when only a limited amount of strain data is available. Finally, the effects of measurement error are discussed.

2. Inverse analysis of pressure distribution

We consider a load (pressure) continuously distributed on the x–y plane: $p = p(x, y)$. Current methods can easily be applied to the three-dimensional case. In the conventional finite element method, a structure is modeled by a mesh and physical fields are approximately represented by nodal values and interpolation functions. Similarly, our method places a mesh of triangular elements on the surface of a structure on which pressure is distributed. The pressure inside each element $p^e(x, y)$ is approximately interpolated as

$$p^e(x, y) = \sum_{i=1}^{3} n_i^e(x, y) p_i^e, \tag{1}$$

where the superscript e denotes a variable or function defined inside an element. p_i^e and $n_i^e(x, y)$ $(i = 1, 2, 3)$ are the nodal pressure values and the linear interpolation functions, respectively.

By superposing Eq. (1), the entire pressure distribution is written as

$$p(x, y) = \sum_{i=1}^{m_p} \hat{p}_i(x, y), \tag{2}$$

where

$$\hat{p}_i(x, y) = N_i(x, y) p_i, \tag{3}$$

m_p is the total number of nodes, and p_i $(i = 1, \ldots, m_p)$ are the nodal values of the pressure. The full-field interpolation function $N_i(x, y)$ $(i = 1, \ldots, m_p)$ is obtained by superposing $n_i^e(x, y)$'s.

We consider only the linear elastic problem. Then, the strain field is expressed as the superposition of strains $\hat{\varepsilon}_i$ which are induced by pressures $\hat{p}_i(x, y)$, and $\hat{\varepsilon}_i$ is proportional to $\hat{p}_i(x, y)$. Thus, by writing $\hat{\varepsilon}_i = \hat{s}'_i \hat{p}_i$, we obtain the following equation for the entire strain field $\varepsilon = \varepsilon(x, y, z)$ of arbitrary components:

$$\varepsilon = \sum_{i=1}^{m_p} \hat{\varepsilon}_i = \sum_{i=1}^{m_p} \hat{s}'_i \hat{p}_i = \sum_{i=1}^{m_p} \hat{s}'_i N_i p_i = \sum_{i=1}^{m_p} s'_i p_i, \tag{4}$$

where $s'_i = \hat{s}'_i N_i$.

Letting ε_j^m $(j = 1, 2, \ldots, m_s)$ be measured strain values (m_s is the number of strain measurements), Eq. (4) gives

$$\varepsilon_j^m = \sum_{i=1}^{m_p} s_{ji} p_i, \quad j = 1, 2, \ldots, m_s, \tag{5}$$

where j is an index of the strain data. This equation shows that s_{ji} is the strain at the location where ε_j^m is measured setting $p_i = 1$ at the i-th node (other $p_k = 0$, $k \neq i$). Thus the matrix s_{ji} is easily obtained by finite element analysis.

We solve Eq. (5) to recover the pressure distribution (p_i, $i = 1, 2, \ldots, m_p$). Note that since $m_s \neq m_p$ in general, the matrix $[s]$ is not square. The most direct technique to solve this problem is to use a generalized inverse matrix $[s^+]$ as follows:

$$p_i = \sum_{j=1}^{m_s} s_{ij}^+ \varepsilon_j^m, \quad i = 1, 2, \ldots, m_p. \tag{6}$$

This method will be employed in the next section.

Another way to solve Eq. (5) is to minimize the error between the measured and estimated strain values. This procedure will be employed in the later part of Section 4 when an aerodynamical restriction is incorporated.

It is emphasized that the effect of structural deformation on the pressure distribution (aeroelastic effect) is not considered in the present study. Incorporation of aeroelastic effects remains an important issue to be addressed in the future.