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Modified Newton, rank-1 Broyden update and rank-2 BFGS update methods in helicopter trim: A comparative study

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ABSTRACT

Nonlinear equations in mathematical physics and engineering are solved by linearizing the equations and forming various iterative procedures, then executing the numerical simulation. For strongly nonlinear problems, the solution obtained in the iterative process can diverge due to numerical instability. As a result, the application of numerical simulation for strongly nonlinear problems is limited. Helicopter aeroelasticity involves the solution of systems of nonlinear equations in a computationally expensive environment. Reliable solution methods which do not need Jacobian calculation at each iteration are needed for this problem. In this paper, a comparative study is done by incorporating different methods for solving the nonlinear equations in helicopter trim. Three different methods based on calculating the Jacobian at the initial guess are investigated.

1. Introduction

The solution of a system of simultaneous nonlinear equations is a cherished topic in numerical analysis. Many relationships in nature are inherently nonlinear; the effects are not in direct proportion to their cause. We can find considerable number of nonlinear problems in engineering [19]. Until the advent of the latest generation of high speed supercomputers, it was highly impractical to use memory intensive methods. However, today's machines are allowing researchers to reconsider memory intensive implicit schemes like Newton's method. The advantage in using these schemes lie in convergence rates. However, these schemes have large CPU time and memory requirements due to the complexity of the Jacobian matrix formation and solution process. One approach is to update select matrix entries only when necessary [14]. This idea lead to a question of exactly how "correct" the Jacobian matrix must be in order to obtain quadratic or better convergence. Hence there was a need for a method that enjoys nearly quadratic convergence rates and is at the same time computationally efficient in the Jacobian matrix formation and inversion process.

Many possibilities exist for improving the performance of these schemes including simplifying the Jacobian matrix formation and solution process [7]. There have been several attempts to integrate the quasi-Newton construction with structural properties of the Jacobian of nonlinear systems which derive from approximations to functional equations [8]. The objectives are to obtain the rapid convergence rates of Newton-like iterative methods simultaneously with a reduction in the computational expense associated with high-dimensional problems. The Jacobian is approximated by a matrix rather than by calculating the first order derivatives. Several Jacobian matrix simplification ideas for Newton method were evaluated in the literature. Of several iterative methods available to solve systems of nonlinear equations, one of the most sophisticated procedures for constructing a new input quantity is the Broyden's method [4]. Although a Broyden rank-1 formula which is employed to update the Jacobian approximation matrix gives good results, it limits the freedom in changing the updated matrix components which may slow the convergence. Hence, a novel modified BFGS rank-2 update method [18] was developed for the nonsymmetric case for which the Jacobian is a rectangular matrix. It provides better convergence over the Broyden rank-1 update method for such cases. In the current paper, all the afore mentioned methods are used for solving the system of nonlinear equations involved in helicopter trim in rotor aeroelastic analysis.

Comprehensive helicopter rotor aeroelastic analysis typically involves three nested iterative numerical schemes to solve for blade response, vehicle trim and free wake inflow distribution on the rotor disk. In general, the word "trim" is used to imply the correct adjustment of aircraft controls, attitude and cargo in order to obtain a desired steady flight condition [23,24,15,25]. For rotorcraft analysis, the concept of trimmed flight implies the periodic dynamic solution to a system of nonlinear equations with unknown parameters (like controls and airframe attitudes), which act as constants and forcing functions in these nonlinear equations. The

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parameters must be adjusted such that this periodic solution satisfies the constraints that enable a desired flight condition. Thus the solution is obtained in controls, attitudes and power required for that flight condition. The accurate calculation of trim is crucial to the determination of flight mechanics and handling qualities [1]. Furthermore, the aeroelastic stability of rotorcraft is strongly influenced by trim settings and periodic trimmed solution [16]. Since the blade equations are highly nonlinear, an accurate trimmed solution is important to predict the response, vibratory loads and airframe vibrations.

Hence to study the behavior of the aircraft (i.e. the helicopter) as the controls are changed, we need to solve the system of nonlinear equations with unknowns (like controls and attitude angles) for each set of the controls, input to the system. In this paper, we look at different methods for solving the coupled nonlinear equations, and propose an efficient and robust approach for the solving the strongly coupled nonlinear equations involved in helicopter trim.

2. Formulation of the problem

The helicopter is represented by a nonlinear model of rotating elastic rotor blades dynamically coupled to a six-degree-of-freedom rigid fuselage. Each blade undergoes flap bending, lag bending, elastic twist and axial displacement. Governing equations are derived using a generalized Hamilton's principle applicable to nonconservative systems [12]:

$$\int_{\psi_1}^{\psi_2} (\delta U - \delta T - \delta W)\, d\psi = 0 \tag{1}$$

Here, δU, δT and δW are virtual strain energy, kinetic energy and virtual work respectively. δU and δT include energy contributions from components that are attached to the blade, e.g., pitch link, lag damper, etc. These equations are based on the work of Hodges and Dowell [12] and include second order geometric nonlinear terms accounting for moderate deflections in the flap bending, lag bending, axial and torsion equations. External aerodynamic forces on the rotor blade contribute to the virtual work variational, δW. The aerodynamic forces and moments are calculated using a free wake inflow distribution and unsteady aerodynamics [13].

Finite element method is used to discretize the governing equations of motion, and allows for accurate representation of complex hub kinematics and nonuniform blade properties [2]. After the finite element discretization, Hamilton's principle is written as

$$\int_{\psi_i}^{\psi_f} \sum_{i=1}^{N} (\delta U_i - \delta T_i - \delta W_i)\, d\psi = 0 \tag{2}$$

Each of the N beam finite elements has 15 degrees of freedom. These degrees of freedom correspond to cubic variations in axial elastic and (flap and lag) bending deflections, and quadratic variation in elastic torsion. Between the elements, there is continuity of slope and displacement for flap and lag bending deflections and continuity of displacements for elastic twist and axial deflections. This element ensures physically consistent linear variations of bending moments and torsion moments and quadratic variations of axial force within the elements. The shape functions used here are Hermite polynomials for lag and flap bending and Lagrange polynomials for axial and torsion deflection and are given in Ref. [2].

Assembling the blade finite element equations and applying boundary conditions results in Eq. (2) becoming

$$\mathbf{M}\ddot{\mathbf{q}}(\psi) + \mathbf{C}\dot{\mathbf{q}}(\psi) + \mathbf{K}\mathbf{q}(\psi) = \mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}, \psi) \tag{3}$$

The nodal displacements $\mathbf{q}$ are functions of time and all nonlinear terms have been moved into the force vector in the right-hand side. The spatial functionality has been removed by using finite element discretization and partial differential equations have been converted to ordinary differential equations. The finite element equations representing each rotor blade are transformed to normal mode space for efficient solution of blade response using the modal expansion. Typically, 6–10 modes are used. The displacements are expressed in terms of normal modes as

$$\mathbf{q} = \boldsymbol{\Phi}\mathbf{p} \tag{4}$$

Substituting Eq. (4) into Eq. (3) leads to normal mode equations having the form

$$\bar{\mathbf{M}}\ddot{\mathbf{p}}(\psi) + \bar{\mathbf{C}}\dot{\mathbf{p}}(\psi) + \bar{\mathbf{K}}\mathbf{p}(\psi) = \bar{\mathbf{F}}(\mathbf{p}, \dot{\mathbf{p}}, \psi) \tag{5}$$

These equations are nonlinear ODEs but their dimensions are much reduced compared to the full finite element equation (3). The normal mode mass, stiffness, damping matrix and force vector are given by

$$\bar{\mathbf{M}} = \boldsymbol{\Phi}^{\mathbf{T}}\mathbf{M}\boldsymbol{\Phi}, \qquad \bar{\mathbf{C}} = \boldsymbol{\Phi}^{\mathbf{T}}\mathbf{C}\boldsymbol{\Phi}, \qquad \bar{\mathbf{K}} = \boldsymbol{\Phi}^{\mathbf{T}}\mathbf{M}\boldsymbol{\Phi}, \qquad \bar{\mathbf{F}} = \boldsymbol{\Phi}^{\mathbf{T}}\mathbf{F} \tag{6}$$

The mode shapes or eigenvectors in Eqs. (4) and (6) are obtained from rotating frequencies of the blade [6]:

$$\mathbf{K_s}\boldsymbol{\Phi} = \omega^2\mathbf{M_s}\boldsymbol{\Phi} \tag{7}$$

The blade normal mode equations in Eq. (5) can be written in the following variational form [17]:

$$\int_{0}^{2\pi} \delta\mathbf{p}^{\mathrm{T}}(\bar{\mathbf{M}}\ddot{\mathbf{p}} + \bar{\mathbf{C}}\dot{\mathbf{p}} + \bar{\mathbf{K}}\mathbf{p} - \bar{\mathbf{F}})\, d\psi = 0 \tag{8}$$

Integrating Eq. (8) by parts, we obtain

$$\int_{0}^{2\pi} \begin{Bmatrix} \delta\mathbf{p} \\ \delta\dot{\mathbf{p}} \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} \bar{\mathbf{F}} - \bar{\mathbf{C}}\dot{\mathbf{p}} - \bar{\mathbf{K}}\mathbf{p} \\ \bar{\mathbf{M}}\dot{\mathbf{p}} \end{Bmatrix} d\psi = \begin{Bmatrix} \delta\mathbf{p} \\ \delta\dot{\mathbf{p}} \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} \bar{\mathbf{M}}\dot{\mathbf{p}} \\ \mathbf{0} \end{Bmatrix} \Bigg|_{0}^{2\pi} \tag{9}$$

Since the helicopter rotor is a periodic system with a time period of one revolution, we have $\dot{\mathbf{p}}(0) = \dot{\mathbf{p}}(\mathbf{2\pi})$. Imposing periodic boundary conditions on Eq. (9) results in the right-hand side becoming zero and yields the following system of first order ordinary differential equations [17]:

$$\int_{0}^{2\pi} \delta\mathbf{y}^{\mathrm{T}}\mathbf{Q}\, d\psi = \mathbf{0} \tag{10}$$

where

$$\mathbf{y} = \begin{Bmatrix} \mathbf{p} \\ \dot{\mathbf{p}} \end{Bmatrix}, \qquad \mathbf{Q} = \begin{Bmatrix} \mathbf{F} - \mathbf{C}\dot{\mathbf{p}} - \mathbf{K}\mathbf{p} \\ \mathbf{M}\dot{\mathbf{p}} \end{Bmatrix} \tag{11}$$

The nonlinear, periodic, ordinary differential equations are then solved for blade steady response using the finite element in time in conjunction with Newton–Raphson method. Discretizing Eq. (6) over N_t time elements around the rotor disk (where $\psi_1 = 0$, $\psi_{N_{t+1}} = 2\pi$) and taking first order Taylor series expansion about the steady state value $\mathbf{y}_0 = [\mathbf{p}_0^{\mathbf{T}}\ \dot{\mathbf{p}}_0^{\mathbf{T}}]^{\mathbf{T}}$ yields the following algebraic equations [17].

$$\sum_{i=1}^{N_t} \int_{\psi_i}^{\psi_{i+1}} \boldsymbol{\delta}\mathbf{y}_i^{\mathbf{T}}\mathbf{Q}_i(\mathbf{y}_0 + \boldsymbol{\Delta}\mathbf{y})\, d\psi = \sum_{i=1}^{N_t} \int_{\psi_i}^{\psi_{i+1}} \boldsymbol{\delta}\mathbf{y}_i^{\mathbf{T}}\big[\mathbf{Q}_i(\mathbf{y}_0) + \mathbf{K}_{ti}(\mathbf{y}_0)\boldsymbol{\Delta}\mathbf{y}\big]\, d\psi = 0 \tag{12}$$