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Surrogate-based parameter optimization and optimal control for optimal trajectory of Halo orbit rendezvous

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ABSTRACT

This work considers the optimization of rendezvous trajectories for spacecraft starting on different z-amplitude Halo orbits. A surrogate-based parameter optimization strategy is proposed for the optimal trajectory of Halo orbit rendezvous in the Sun–Earth system. The optimal rendezvous problem is transformed into an optimal control problem which fixes the initial flight time and the time of flight for the rendezvous trajectory. Further, the initial flight time and the time of flight are taken as design variables for parameter optimization under the objective of minimum fuel consuming. Since the precise optimization model is typically time consuming and computational expensive, the surrogate model is constructed using data drawn from the precise model, and provides fast approximation of the objective at new design points. Therefore, the surrogate model is feasible and employed for Halo orbit rendezvous in this paper. Numerical simulations show that the surrogate-based parameter optimization takes advantages over the global traversal method and the global optimization method (genetic algorithm). At last, the influences of the different relative z-amplitude Halo orbits for optimal trajectory of Halo orbit rendezvous have been studied by employing the surrogate-based parameter optimization strategy in the numerical simulations.

1. Introduction

The exploration of libration point space environment has been accompanied by a rich and growing literature on the design, modeling, and control of a variety of orbits in the circular restricted there-body problem (CRTBP) [14,5,26]. Because the special position of libration point [3], fuel-efficient transfer trajectories could be used in future lunar missions, such as south pole communications satellite architectures [18]. One important problem is to design the connections trajectories between escape trajectories from Earth Halo orbits and capture trajectories to a target planet's Halo orbit for interplanetary transfer [15]. Another important problem is that the design of optimal rendezvous trajectories between the different Halo orbits. Indeed, the rendezvous trajectories for spacecraft starting on different z-amplitude Halo orbits are very different from the spacecraft rendezvous within the frame of two-body dynamics.

Different with the Kepler orbits, the Halo orbits are inherently unstable, and a spacecraft with a little deviation from the accurate initial conditions will depart from the desired orbits after a period of time. Within two-body dynamics, Kepler orbits that are significantly different in size (semi-major axis) will have significantly different orbital periods, and the relative velocity of the two spacecraft will vary significantly. These statements do not apply to Halo orbits of different size (z-amplitude). Increasing the z-amplitude of Halo orbits does not significantly affect the orbital periods [25]. There are some previous works that are studied with the similar problem of Halo orbit rendezvous. Ref. [25] employed variable-specific-impulse technology to design optimal rendezvous trajectories between Halo orbits. In Ref. [13], the minimum propellant optimal maneuvers of space vehicles equipped with low-thrust propulsion installation for rendezvous in the Earth–Moon system are examined. Similar, the nonlinear problem of the optimal libration points rendezvous in Earth–Moon system is examined in Ref. [12].

From the above literatures, the parameter optimization of Halo orbit rendezvous has been emphasized as an important problem. Generally, this problem is solved based on the precise optimization model using global optimization method or local optimization method, such as the genetic algorithm [9] or the sequential quadratic programming (SQP) method [16]. Ref. [1] takes the total flight time and the summation of impulsive maneuvers as the objective functions to be minimized, so a multi-objective genetic optimization is employed for solving this problem in the restricted four-body problem. Similarly, by applying the particle swarm optimization algorithm, trajectory design for the Moon departure libration point mission is implemented in full ephemeris model [2]. The popularity of evolutionary algorithm lies in ease of implementation and their ability to converge close to the global optimal design.

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However, evolutionary algorithms typically require thousands of function evaluations and also require significant user interaction to converge to a global solution. This progress may take many hours of computer time.

Two categories of techniques have been proposed to tackle the efficiency issue of evolutionary search methods; the first type is focused on devising more efficient variants of the canonical algorithms, the second type involves using a surrogate model which is a kind of approximation in lieu of the exact and often expensive function evaluations [19]. Surrogate-based optimization (SBO) has been shown to be an effective approach for the design of computational expensive models such as many optimizations in aerospace and aerospace system [22,30,6,11]. SBO has been suggested as an effective approach for the design with time-consuming computer models. The basic concept of SBO is that the direct optimization of the computationally expensive model is replaced by an iterative process that involves the creation, optimization and updating of a fast and analytically tractable surrogate model [11]. For example, in order to reduce time expenditure for optimization in system and control, a weighted average surrogate approach is applied to helicopter rotor blade vibration reduction [8]. By using three test problems: an Earth–Mars transfer orbit problem, the analytic Shekel function, and a low Earth orbit three-satellite constellation design problem, datascape, Kriging and second order regression surrogate modeling techniques are compared on model accuracy, computational efficiency, robustness, etc. [7].

In our previous work [21], the problem of formation flying on Halo orbit has been studied by optimal periodic controller. The main attention has been focused on the keeping of formation configuration. Different from Ref. [21], the problem of optimization trajectory of Halo orbit rendezvous is considered that the rendezvous of two spacecrafts starting on Halo orbits of different z-amplitudes by using continuous low thrust in this study. The chaser spacecraft leaves from a Halo orbit to capture a target spacecraft on another Halo orbit. Meanwhile, the target spacecraft is assumed to be nominally coasting along its prescribed Halo orbits. The initial epoch time of flight and the time of flight for rendezvous process are allowed to be free and should be optimized. The nonlinear optimal control for minimum fuel consuming of rendezvous process is solved under the given pair of the initial flight time and the time of flight. Therefore, many different solutions can be generated with different independent parameters. The independent parameters of the initial flight time and the time of flight for rendezvous would be optimized based on surrogate model to increase the efficiencies and decrease the expensive computation.

The paper is organized as follows: First, the nonlinear dynamical model for Halo orbit rendezvous is introduced based on the CRTBP in Section 2. Subsequently, a nonlinear optimal control algorithm is proposed based on the discrete dual variables for the missions of Halo orbit rendezvous in Section 3. In Section 4, a surrogate-based parameter optimization strategy is given for decrease the computational expensiveness and time. Finally, the surrogate-based parameter optimization strategy for searching optimal trajectory of Halo orbit rendezvous is evaluated in numerical simulations and the influence of orbital amplitude has been discussed in Section 5.

2. Dynamical model

The CRTBP model is utilized to investigate the motion surrounding the collinear libration point. It is assumed that the mass m of the spacecraft is insignificant compared to the mass m_1 of the Sun and the mass m_2 of the Earth. Hence, the orbital motion of the two primary bodies (i.e., the Sun and the Earth) is not affected by the third body spacecraft. It is further assumed that the two primary bodies rotate about their barycenter in circular orbits under the constant angular velocity n. A rotating reference frame (O, X, Y, Z) is defined with its origin at the barycenter of the Sun–Earth system. The X unit vector is directed from the Sun towards the Earth. The Y unit vector is defined as normal to the X vector in the plane of the primary orbit and along the prograde rotational direction. The Z unit vector then completes the right-handed frame and is thus normal to the plane of the primaries' orbit [27]. If a spacecraft is located by a position vector with a base point at the barycenter using coordinates X, Y and Z with respect to the rotating frame, the well-known equations of motion for CRTBP can be written in the dimensionless form [24]:

$$\ddot{X} - 2\dot{Y} + \bar{U}_X = 0 \tag{1}$$

$$\ddot{Y} + 2\dot{X} + \bar{U}_Y = 0 \tag{2}$$

$$\ddot{Z} + \bar{U}_Z = 0 \tag{3}$$

and

$$\bar{U}(X, Y, Z) = -\frac{1}{2}\left(X^2 + Y^2\right) - \frac{1-\mu}{r_1} - \frac{\mu}{r_2} - \frac{1}{2}\mu(1-\mu) \tag{4}$$

where, the dot represents time derivative in the rotating frame, μ is the ratio of the Earth mass to the sum of the masses of both the Earth and the Sun and $r_1 = \sqrt{(X+\mu)^2 + Y^2 + Z^2}$, $r_2 = \sqrt{(X-(1-\mu))^2 + Y^2 + Z^2}$.

It can be conveniently to transfer the reference frame from the barycenter of the Sun–Earth system to the L_2 point, since this paper mainly discusses the problem of collinear libration orbits around the L_2 point of the Sun–Earth system. The relationship between the (O, X, Y, Z) reference frame and the (L_2, x, y, z) reference frame is as follows:

$$x = \frac{(X - 1 + \mu - \gamma)}{\gamma}, \qquad y = \frac{Y}{\gamma}, \qquad z = \frac{Z}{\gamma} \tag{5}$$

where, the distance between the Earth and the L_2 point denoted by $\gamma = 1.00782 \times 10^{-2}$ as the new unit of length. Based on Eq. (5), the motion equations of CRTBP (1)–(3) can be rewritten as follows:

$$\ddot{x} - 2\dot{y} - x = -\frac{(1-\mu)(x+1+1/\gamma)}{\gamma^3 d_1^3} - \frac{\mu(x+1)}{\gamma^3 d_2^3} + \frac{1-\mu+\gamma}{\gamma} \tag{6}$$

$$\ddot{y} + 2\dot{x} - y = -\frac{(1-\mu)y}{\gamma^3 d_1^3} - \frac{\mu y}{\gamma^3 d_2^3} \tag{7}$$

$$\ddot{z} = -\frac{(1-\mu)z}{\gamma^3 d_1^3} - \frac{\mu z}{\gamma^3 d_2^3} \tag{8}$$

where,

$$d_1 = \sqrt{(x+1+1/\gamma)^2 + y^2 + z^2} \quad \text{and}$$

$$d_2 = \sqrt{(x+1)^2 + y^2 + z^2}.$$

It is well known that the collinear libration points have families of periodic and quasi-periodic unstable orbits. Halo orbits are special periodic orbits around the collinear libration points, as shown in Fig. 1. How to find the optimal trajectory of Halo orbit rendezvous from the low (high) to the high (low) Halo orbits is the main interest in this paper.

The determination of Halo orbits can be solved by a differential correction algorithm [23]. The initial conditions of these Halo orbits can be given as $\mathbf{x}_0 = [x(0), y(0), z(0), \dot{x}(0), \dot{y}(0), \dot{z}(0)]^{\mathrm{T}}$. Because of the orbit symmetry with respect to the $(x–z)$ plane, we obtain the initial condition $y(0) = 0$, $\dot{x}(0) = 0$ and $\dot{z}(0) = 0$. Then a differential correction algorithm can be applied to find the other