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Robust Kalman filtering for discrete-time nonlinear systems with parameter uncertainties

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ABSTRACT

This paper focuses on the robust Kalman filtering problem for discrete-time nonlinear systems with norm-bound parameter uncertainties. An explicit solution to the robust Kalman filtering problem is presented based on a Riccati equation approach. A new Riccati equation is derived in the presence of both the parameter uncertainties and the linearization errors. The proposed filter is illustrated by simulation on a pulsar positioning system (PPS) in comparison with the standard extended Kalman filter (EKF) and the robust H_∞ filter (RHF). To facilitate the application of the robust filter, a heuristic method is proposed to estimate the bounds of the model parameter uncertainties for the considered PPS.

1. Introduction

The Kalman filter (KF) and the extended Kalman filter (EKF) have important applications in many fields, such as signal processing, data fusion and target tracking [31,13,29]. For the design of the KF, accurate system model is required. In the presence of model uncertainties, the performance of the standard KF may be degraded. This has motivated many studies of robust filtering (see, e.g., [26,22,10,7,12,21,8,20,27] and references therein).

The robust Kalman filtering (RKF) algorithms have been designed for linear models with uncertainties [33,32,11,24,25]. The performance objective of the RKF is to ensure a minimum possible upper bound on the estimation error covariance. Two popular approaches used to develop the RKF algorithms are the Riccati equation approach [33,32,11] and the linear matrix inequality (LMI) approach [24,25]. More research references on this topic can be found in [2,17] for continuous-time systems and [15,23,6] for discrete-time systems. These works are, however, restricted to the case of linear systems. The RKF problem for nonlinear systems has gained less attention. This situation motivates our present investigation.

This paper proposes an RKF algorithm for discrete-time nonlinear systems with norm-bound parameter uncertainties. A pseudo-linearization technique is adopted to derive the Riccati equation, such that the proposed algorithm will be simple and familiar to the practicers. Both the parameter uncertainties and the linearization errors are taken into consideration during the derivation. We refer to our algorithm as the robust extended Kalman filter (REKF) as its structure resembles that of the EKF. In [16], an extended robust H_∞ filter is proposed for continuous-time nonlinear systems with uncertainties described by an integral quadratic constraint (IQC). See also [14] and [9] for other related works. Different from this kind of robust filters, no IQC or SQC (sum quadratic constraint) is required for the proposed REKF. Another robust filter proposed for nonlinear system is the robust EKF [5]. As the algorithm is designed by using the H_∞ technique, it is proper to call it as the "extended H_∞ filter".

A problem of the RKF is still unsolved, which is the design of the bounds of the uncertainties that appear in the Riccati equation (or LMI). In the above mentioned literature, there is a common assumption that the bounds of the uncertainties are known. However, it may not be realistic in practice. Such an assumption does not only limit the applicable domain of the relevant algorithms, but also often results in conservative designs. In this paper, a heuristic method is proposed to estimate the bounds of the uncertain model parameters in the observation equation of the pulsar positioning system (PPS).

The PPS is an autonomous celestial-based navigation system for the spacecrafts. From 1999 through 2000, the U.S. Naval Research Laboratory's (NRL) unconventional stellar aspect (USA) experiment onboard the Advanced Research and Global Observation Satellite (ARGOS) was performed to demonstrate the feasibility of the X-ray pulsar-based navigation [28]. It is reported in [19] that the positioning accuracy of the PPS is on the order of 2 km. Due to the limitation of current technology, the model parameters related to the pulsar position information are not determined to high accuracy. The parameter uncertainties may degrade the positioning performance. In our prior works [30], the difference technique is

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investigated for satellite constellation to eliminate the common error terms caused by the parameter uncertainties. In this paper, the robust filtering technique is adopted to suppress the unfavorable effect of the parameter uncertainties.

This paper is organized as follows. In Section 2, the problem to be solved is presented, and Section 3 derives the REKF. A brief description of the PPS and the method to estimate the bounds of the uncertainties are given in Section 4. Comparisons with the EKF and the robust H_∞ filter (RHF) are made in Section 5. The conclusion is drawn in Section 6.

Notation: $\boldsymbol{R}^n$ denotes the n-dimensional Euclidean space, $\boldsymbol{R}^{n\times m}$ is the set of $n \times m$ real matrices. E(.) denotes the mathematical expectation. The superscript "T" denotes the transpose. The inequality $\boldsymbol{X} \geqslant 0$ means that the matrix $\boldsymbol{X}$ is symmetric and positive semi-definite, and $\boldsymbol{X} \geqslant \boldsymbol{Y}$ means $\boldsymbol{X} - \boldsymbol{Y} \geqslant 0$. Similar definitions apply to symmetric positive/negative definite matrices. δ_{kj} denotes the Kronecker delta function. $\mathcal{N}(\boldsymbol{x}, \boldsymbol{P})$ denotes a normal distribution with mean $\boldsymbol{x}$ and covariance $\boldsymbol{P}$. The matrix square root of positive definite matrix $\boldsymbol{P}$ means a matrix $\boldsymbol{A} = \sqrt{\boldsymbol{P}}$ (or $\boldsymbol{A} = \boldsymbol{P}^{1/2}$) such that $\boldsymbol{P} = \boldsymbol{A}\boldsymbol{A}^T$. In this paper, the inequalities with random variables hold with probability one.

2. Problem statement

Consider the following uncertain nonlinear system

$$\boldsymbol{x}_k = f(\boldsymbol{x}_{k-1}) + \boldsymbol{A}_k \Delta_{Ak} \boldsymbol{E}_{Ak} \boldsymbol{x}_{k-1} + \boldsymbol{w}_k \tag{1}$$

$$\boldsymbol{y}_k = h(\boldsymbol{x}_k) + \boldsymbol{C}_k \Delta_{Ck} \boldsymbol{E}_{Ck} \boldsymbol{x}_k + \boldsymbol{v}_k \tag{2}$$

where $\boldsymbol{x}_k \in \boldsymbol{R}^l$ is the state at time k, $\boldsymbol{y}_k \in \boldsymbol{R}^m$ is the measurement, $f(\boldsymbol{x}_{k-1})$ and $h(\boldsymbol{x}_k)$ are nonlinear system dynamic and observation models, respectively, which have bounded derivatives. $\boldsymbol{w}_k \in \boldsymbol{R}^l$ and $\boldsymbol{v}_k \in \boldsymbol{R}^m$ are uncorrelated white noises with zero means and known covariance matrices

$$\mathrm{E}\left\{\begin{bmatrix} \boldsymbol{w}_k \\ \boldsymbol{v}_k \end{bmatrix} \begin{bmatrix} \boldsymbol{w}_j^T & \boldsymbol{v}_j^T \end{bmatrix}\right\} = \begin{bmatrix} \boldsymbol{Q}_k \delta_{kj} & 0 \\ 0 & \boldsymbol{R}_k \delta_{kj} \end{bmatrix}. \tag{3}$$

$\boldsymbol{A}_k \Delta_{Ak} \boldsymbol{E}_{Ak} \boldsymbol{x}_{k-1}$ and $\boldsymbol{C}_k \Delta_{Ck} \boldsymbol{E}_{Ck} \boldsymbol{x}_k$ represent the parameter uncertainties, where $\boldsymbol{A}_k$ and $\boldsymbol{C}_k$ are known scaling matrices of appropriate dimensions. $\Delta_{Ak} \in \boldsymbol{R}^{l\times l}$ and $\Delta_{Ck} \in \boldsymbol{R}^{l\times l}$ are unknown matrices satisfying

$$\Delta_{Ak}\Delta_{Ak}^T \leqslant \boldsymbol{I}, \qquad \Delta_{Ck}\Delta_{Ck}^T \leqslant \boldsymbol{I}. \tag{4}$$

$\boldsymbol{E}_{Ak} \in \boldsymbol{R}^{l\times l}$ and $\boldsymbol{E}_{Ck} \in \boldsymbol{R}^{l\times l}$ are known matrices which can be set as

$$\boldsymbol{E}_{Ak} = \boldsymbol{I}, \qquad \boldsymbol{E}_{Ck} = \boldsymbol{I}. \tag{5}$$

Let $\hat{\boldsymbol{x}}_k$ denote the estimate of $\boldsymbol{x}_k$. The objective of the paper is to design a robust filter for the considered uncertain nonlinear system. Specifically, given the measurement $\boldsymbol{y}_i$ $(0 \leqslant i \leqslant k)$, design an estimator of the form

$$\text{Prediction:} \quad \hat{\boldsymbol{x}}_{k|k-1} = f(\hat{\boldsymbol{x}}_{k-1}) \tag{6}$$

$$\text{Update:} \quad \hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k \tilde{\boldsymbol{y}}_k \tag{7}$$

where $\tilde{\boldsymbol{y}}_k = \boldsymbol{y}_k - h(\hat{\boldsymbol{x}}_{k|k-1})$ is the innovation, $\boldsymbol{K}_k$ is filter gain to be determined such that the covariance of the estimation error

$$\tilde{\boldsymbol{x}}_k = \boldsymbol{x}_k - \hat{\boldsymbol{x}}_k \tag{8}$$

is guaranteed to be smaller than a certain bound, i.e., for a given matrix Σ_k, the estimation error satisfies

$$\mathrm{E}\big(\tilde{\boldsymbol{x}}_k \tilde{\boldsymbol{x}}_k^T\big) \leqslant \Sigma_k. \tag{9}$$

Furthermore, we will minimize the error bound Σ_k and obtain an optimized filter eventually. Note that the structure of the standard EKF [5] is adopted for the design of the REKF.

From (1) and (6), the dynamic equation of the prediction error

$$\tilde{\boldsymbol{x}}_{k|k-1} = \boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k-1} \tag{10}$$

is described by

$$\tilde{\boldsymbol{x}}_{k|k-1} = f(\boldsymbol{x}_{k-1}) - f(\hat{\boldsymbol{x}}_{k-1}) + \boldsymbol{A}_k \Delta_{Ak} \boldsymbol{E}_{Ak} \boldsymbol{x}_{k-1} + \boldsymbol{w}_k.$$

Instead of using the following classical approximation [10]:

$$\tilde{\boldsymbol{x}}_{k|k-1} \approx \boldsymbol{F}_k \tilde{\boldsymbol{x}}_{k-1} + \boldsymbol{A}_k \Delta_{Ak} \boldsymbol{E}_{Ak} \boldsymbol{x}_{k-1} + \boldsymbol{w}_k$$

where $\boldsymbol{F}_k = \frac{\partial f}{\partial \boldsymbol{x}}\big|_{\boldsymbol{x}=\hat{\boldsymbol{x}}_{k-1}}$, we introduce an unknown time-varying matrix $\Delta_{Bk} \in \boldsymbol{R}^{l\times l}$ that satisfies

$$\Delta_{Bk}\Delta_{Bk}^T \leqslant \boldsymbol{I} \tag{11}$$

and a known scaling matrix $\boldsymbol{B}_k \in \boldsymbol{R}^{l\times l}$ to model errors due to the first order linearization technique, so that we obtain the following exact equality:

$$\tilde{\boldsymbol{x}}_{k|k-1} = (\boldsymbol{F}_k + \boldsymbol{B}_k \Delta_{Bk} \boldsymbol{E}_{Bk}) \tilde{\boldsymbol{x}}_{k-1} + \boldsymbol{A}_k \Delta_{Ak} \boldsymbol{E}_{Ak} \boldsymbol{x}_{k-1} + \boldsymbol{w}_k. \tag{12}$$

The unknown matrix $\boldsymbol{E}_{Bk} \in \boldsymbol{R}^{l\times l}$ is set as

$$\boldsymbol{E}_{Bk} = \Big(\sqrt{\big(\tilde{\boldsymbol{x}}_{k-1}\tilde{\boldsymbol{x}}_{k-1}^T\big)^{-1}}\Big)^T \tag{13}$$

to facilitate the following deductions. The uncertainty term Δ_{Bk}, together with $\boldsymbol{B}_k$ and $\boldsymbol{E}_{Bk}$, take into account the linearization errors in the model matrix $\boldsymbol{F}_k$, i.e.,

$$\boldsymbol{B}_k \Delta_{Bk} \boldsymbol{E}_{Bk} \tilde{\boldsymbol{x}}_{k-1} = f(\boldsymbol{x}_{k-1}) - f(\hat{\boldsymbol{x}}_{k-1}) - \boldsymbol{F}_k \tilde{\boldsymbol{x}}_{k-1}. \tag{14}$$

For physical processes with finite energy, as the state $\boldsymbol{x}_k$ is bounded, and the magnitude of the estimate $\hat{\boldsymbol{x}}_k$ can be controlled, it is reasonable to assume that the prediction error is scaled by the matrix $\boldsymbol{B}_k$. Similar formulations for the linearization errors have been used for the design of the nonlinear robust filter (see, e.g. [4]) and the stability analysis of the EKF (see, e.g. [3]). Generally, if $\boldsymbol{B}_k$ is set to be large enough, there exists appropriate matrix Δ_{Bk} such that (11) is satisfied.

Similarly, we obtain the exact formulation of the innovation

$$\tilde{\boldsymbol{y}}_k = (\boldsymbol{H}_k + \boldsymbol{D}_k \Delta_{Dk} \boldsymbol{E}_{Dk}) \tilde{\boldsymbol{x}}_{k|k-1} + \boldsymbol{C}_k \Delta_{Ck} \boldsymbol{E}_{Ck} \boldsymbol{x}_k + \boldsymbol{v}_k \tag{15}$$

where $\boldsymbol{H}_k = \frac{\partial h}{\partial \boldsymbol{x}}\big|_{\boldsymbol{x}=\hat{\boldsymbol{x}}_{k|k-1}}$, the unknown time-varying matrix $\Delta_{Dk} \in \boldsymbol{R}^{l\times l}$ that satisfying

$$\Delta_{Dk}\Delta_{Dk}^T \leqslant \boldsymbol{I} \tag{16}$$

together with the known scaling matrix $\boldsymbol{D}_k \in \boldsymbol{R}^{m\times l}$ and the unknown matrix $\boldsymbol{E}_{Dk} \in \boldsymbol{R}^{l\times l}$ that satisfying

$$\boldsymbol{E}_{Dk} = \Big(\sqrt{\big(\tilde{\boldsymbol{x}}_{k|k-1}\tilde{\boldsymbol{x}}_{k|k-1}^T\big)^{-1}}\Big)^T \tag{17}$$

take into account the linearization errors.

Combining (7), (8), (10) and (15) yields the dynamic equation of the estimation error

$$\begin{aligned}\tilde{\boldsymbol{x}}_k = {} & (\boldsymbol{I} - \boldsymbol{K}_k \boldsymbol{H}_k - \boldsymbol{K}_k \boldsymbol{D}_k \Delta_{Dk} \boldsymbol{E}_{Dk}) \tilde{\boldsymbol{x}}_{k|k-1} \\ & - \boldsymbol{K}_k \boldsymbol{C}_k \Delta_{Ck} \boldsymbol{E}_{Ck} \boldsymbol{x}_k - \boldsymbol{K}_k \boldsymbol{v}_k.\end{aligned} \tag{18}$$

For later use, the covariance matrices of the prediction error $\tilde{\boldsymbol{x}}_{k|k-1}$ and the estimation error $\tilde{\boldsymbol{x}}_k$ are defined as

$$\boldsymbol{P}_{k|k-1} = \mathrm{E}\big(\tilde{\boldsymbol{x}}_{k|k-1}\tilde{\boldsymbol{x}}_{k|k-1}^T\big), \qquad \boldsymbol{P}_k = \mathrm{E}\big(\tilde{\boldsymbol{x}}_k \tilde{\boldsymbol{x}}_k^T\big). \tag{19}$$

The problem stated here differs from those considered in the literature (e.g., [32,11,24,25,6]) in that both the parameter uncertainties described by $\boldsymbol{A}_k \Delta_{Ak} \boldsymbol{E}_{Ak} \boldsymbol{x}_{k-1}$ and $\boldsymbol{C}_k \Delta_{Ck} \boldsymbol{E}_{Ck} \boldsymbol{x}_k$, and the linearization errors described by $\boldsymbol{B}_k \Delta_{Bk} \boldsymbol{E}_{Bk} \tilde{\boldsymbol{x}}_{k-1}$ and $\boldsymbol{D}_k \Delta_{Dk} \boldsymbol{E}_{Dk} \tilde{\boldsymbol{x}}_{k|k-1}$ are taken into consideration.