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Nonlinear static analysis of smart laminated composite plate

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ABSTRACT

In this paper, a third order shear deformation theory is used to study the behavior of laminated smart composite plate with magnetostrictive layers, while accounting for geometric nonlinearity in the von-Karman sense. A coupled analysis of magnetostrictive material is presented where Terfenol-D is used as a magnetostrictive material to control the responses of the laminated smart composite plates. A C^0 finite element formulation is proposed for this purpose. The accuracy of the present formulation has been shown by comparison of the present results with those available in the literature. Some new results are also presented for future research in this field of study.

1. Introduction

Smart materials have the ability to respond to changes in stress, strain, displacement, velocity, acceleration, electrical, thermal or other mechanical change of a structure through changes in their properties in a controlled manner to maintain a desirable and satisfactory performance. So, smart materials are quite popular materials to be used as both sensors and actuators in biomedical, aerospace structures, automotive and machine tool industries, flight control, etc. [5,13,21,8]. The most common smart materials are piezoelectric and electrostrictive materials, shape memory alloys, magnetostrictive materials, electro and magneto rheological fluids, etc. When these smart materials are bonded or embedded in conventional structures made of isotropic or composite materials to enhance its performance and capabilities then the structures are known to be smart structures having the capability of sensing, actuating and processing or controlling.

A magnetostrictive material is one of the above mentioned smart materials which undergo dimensional change when exposed to a magnetic field and are capable of changing their magnetic state in response to stresses [25]. This property was first observed by James P. Joule in 1842 [14] when he noted that a sample of iron changes in length when magnetized by a magnetic field. These properties of magnetostrictive materials are used effectively to create actuators and sensors that deliver improved performance over other smart materials. One of the most popular and commercially available magnetostrictive materials is Terfenol-D, which produces relatively low strains and moderate forces over a wide frequency range [6,7]. For its applications in smart laminated structures it is very important to know the interaction between its constituents layers i.e., the magnetostrictive layers and the laminated composites. In this aspect, there exist a number of investigations [10,12,20] which studied the material properties of magnetostrictive material Terfenol-D with regard to its static and dynamic applications. Anjanaappa and Bi [2,3] investigated the feasibility of using embedded magnetostrictive mini actuators for smart structural applications. Beside, the mechanical behavior of smart structures embedded with magnetostrictive layers is influenced by the coupling between magnetic and mechanical effects [4] and moreover, it is necessary to consider geometric and material nonlinearities.

It is well-known fact that the effect of shear deformation is very much important in laminated composite plate due to their very low transverse shear modulus compared to the in-plane modulus and thus, it is very important to consider this effect in the analysis of laminated composites. In order to take this into account a number of plate theories are derived [27]. Among them the commonly used theories are the classical laminated plate theory (CLPT) [31,30] and the first order shear deformation theory (FSDT) [32,33] and higher order shear deformation theories (HSDT) [26,16,23]. All these theories provide approximate solutions for a problem. There exist some three-dimensional (3D) elasticity solutions [22,29] which provide the most accurate solutions for particular types of problems but the availability of these are scanty. Therefore, the use of various plate theories is quite usual practice. Kant and Swaminathan [17] presented analytical formulation of the static analysis of simply supported composite and sandwich plates using various displacement models based on higher order refined shear deformation theory. Using third order shear deformation the-

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ory (TSDT), Aagaah et al. [1] studied deformations of a laminated composite plate subjected to mechanical loads and derived linear dynamic equations for a rectangular multi-layered composite plate. Pradhan et al. [24] presented an analytical solution of vibration control for simply supported laminated plates using first order shear deformation theory. Using a unified plate theory Lee et al. [19] studied control of transient response of laminated composite plates with integrated smart layers (Terfenol-D) used as sensors and actuators. The assumed theory includes the features of classical, first order and third order plate theories. Ghosh and Gopalakrishnan [9] presented a finite element formulation for linear coupled analysis of composite laminate with embedded magnetostrictive layer acting as both sensor and actuator and they showed the ply sequence to have prominent effects on the overall response due to coupling. The above mentioned studies do not consider either geometric or material nonlinearities in the analysis of laminated structures. In this direction, Huai and Hui [11] derived a simple higher order theory for nonlinear bending of generally laminated composite rectangular plates. Kant and Kommineni [15] presented a refined higher order shear deformation theory for the linear and geometrically nonlinear finite element analysis of fiber reinforced composite and sandwich laminates. Lee and Reddy [18] studied deflection control of the laminated composite plate using third order shear deformation theory. The effect of geometric nonlinearity was taken in the von-Karman sense to study its influence on the static and dynamic response of the laminate using finite element method. Terfenol-D was taken as embedded magnetostrictive layers in the plate to control the plate response.

It is evident from the above discussion that the studies on the nonlinear static analysis of the smart composite plate embedded with magnetostrictive materials are limited. So, in the present paper it is attempted to study the nonlinear static behavior of the composite plates embedded with magnetostrictive materials based on the third order shear deformation theory taking into account the geometric nonlinearity in von-Karman sense. Four different displacement fields have been assumed to bring out the effects of shear deformation. For this purpose a C^0 finite element formulation is developed and applied for the static analysis of laminated composite plates and the same has also been applied to smart composite plate embedded with magnetostrictive layers. In fact, the present study simply extends the work of Lee and Reddy [18] incorporating the coupled analysis of the composite laminate with embedded magnetostrictive layers acting as both sensors and actuators. Numerical results for four assumed displacement fields are presented to compare the deflection response for various parameters like different lamination schemes, loading conditions, and boundary conditions.

2. Mathematical formulation

2.1. Displacement models

In the present study, the displacement field within the laminate is assumed to be based on third order shear deformation theory. In this, the in-plane displacements are expanded as cubic functions of the thickness coordinate while the transverse displacement is assumed to vary in two different ways along the thickness direction and accordingly two different displacement models are obtained. In one model transverse displacement varies linearly through the plate thickness (Model 1.1) [17] and in the other model it is independent of the plate thickness (Model 2.1) [11]. In addition to the above assumed displacement models, if traction free boundary conditions are applied on top and bottom surfaces of the plate, above-mentioned displacement models as defined (Model 1.1 and Model 2.1) are modified, i.e., Model 1.2 [15] obtained from Model 1.1 and Model 2.2 [1] obtained from Model 2.1. The assumed displacement models are:

Model 1.1:

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\phi_x(x,y) + z^2\theta_x(x,y) + z^3\lambda_x(x,y), \\ v(x,y,z) &= v_0(x,y) + z\phi_y(x,y) + z^2\theta_y(x,y) + z^3\lambda_y(x,y), \\ w(x,y,z) &= w_0(x,y) + zw_1(x,y), \end{aligned} \tag{1}$$

with

$$\begin{aligned} &(u_0, v_0, w_0) = (u, v, w)_{z=0}, \\ &\phi_x = \left(\frac{\partial u}{\partial z}\right)_{z=0}, \qquad \phi_y = \left(\frac{\partial v}{\partial z}\right)_{z=0}, \\ &\theta_x = \frac{1}{2}\left(\frac{\partial^2 u}{\partial z^2}\right)_{z=0}, \qquad \theta_y = \frac{1}{2}\left(\frac{\partial^2 v}{\partial z^2}\right)_{z=0}, \\ &\lambda_x = \frac{1}{6}\left(\frac{\partial^3 u}{\partial z^3}\right)_{z=0}, \qquad \lambda_y = \frac{1}{6}\left(\frac{\partial^3 v}{\partial z^3}\right)_{z=0}. \end{aligned}$$

Model 1.2:

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\phi_x(x,y) + C_1z^2\theta_{2x}(x,y) \\ &\quad + z^3C_2\left[\phi_x(x,y) + \theta_{1x}(x,y)\right], \\ v(x,y,z) &= v_0(x,y) + z\phi_y(x,y) + C_1z^2\theta_{2y}(x,y) \\ &\quad + z^3C_2\left[\phi_y(x,y) + \theta_{1y}(x,y)\right], \\ w(x,y,z) &= w_0(x,y) + zw_1(x,y), \end{aligned} \tag{2}$$

with

$$\begin{aligned} &\phi_x = \left(\frac{\partial u}{\partial z}\right)_{z=0}, \qquad \phi_y = \left(\frac{\partial v}{\partial Z}\right)_{z=0}, \\ &\theta_{1x} = \left(\frac{\partial w_0}{\partial x}\right)_{z=0}, \qquad \theta_{2x} = \left(\frac{\partial w_1}{\partial x}\right)_{z=0}, \\ &\theta_{1y} = \left(\frac{\partial w_0}{\partial y}\right)_{z=0}, \qquad \theta_{2y} = \left(\frac{\partial w_1}{\partial y}\right)_{z=0}, \\ &C_1 = -\frac{1}{2}, \qquad C_2 = -\frac{4}{3h^2}. \end{aligned}$$

Model 2.1:

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\phi_x(x,y) + z^2\theta_x(x,y) + z^3\lambda_x(x,y), \\ v(x,y,z) &= v_0(x,y) + z\phi_y(x,y) + z^2\theta_y(x,y) + z^3\lambda_y(x,y), \\ w(x,y,z) &= w_0(x,y), \end{aligned} \tag{3}$$

with

$$\begin{aligned} &(u_0, v_0, w_0) = (u, v, w)_{z=0}, \\ &\phi_x = \left(\frac{\partial u}{\partial z}\right)_{z=0}, \qquad \phi_y = \left(\frac{\partial v}{\partial z}\right)_{z=0}, \\ &\theta_x = \frac{1}{2}\left(\frac{\partial^2 u}{\partial z^2}\right)_{z=0}, \qquad \theta_y = \frac{1}{2}\left(\frac{\partial^2 v}{\partial z^2}\right)_{z=0}, \\ &\lambda_x = \frac{1}{6}\left(\frac{\partial^3 u}{\partial z^3}\right)_{z=0}, \qquad \lambda_y = \frac{1}{6}\left(\frac{\partial^3 v}{\partial z^3}\right)_{z=0}. \end{aligned}$$

Model 2.2:

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\phi_x(x,y) - z^3C_1\left[\phi_x(x,y) + \theta_x(x,y)\right], \\ v(x,y,z) &= v_0(x,y) + z\phi_y(x,y) - z^3C_1\left[\phi_y(x,y) + \theta_y(x,y)\right], \\ w(x,y,z) &= w_0(x,y), \end{aligned} \tag{4}$$