Aerospace Science and Technology 14 (2010) 38–48

Contents lists available at ScienceDirect

Aerospace Science and Technology

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An analysis of maximum range cruise including wind effects

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ARTICLE INFO

Article history:
Received 12 March 2008
Received in revised form 14 September 2009
Accepted 17 November 2009
Available online 20 November 2009

Keywords:
Aircraft performance
Cruise regime
Maximum range
Wind effects

ABSTRACT

In this article the maximum range cruise is analyzed. Wind effects are included in the analysis, taking into account the variation of wind speed with altitude (crosswinds are ignored). The optimal control laws that lead to maximum range are analyzed (unconstrained case); constrained regimes (constant-altitude cruise or constant-Mach cruise) are also analyzed. The maximum range in the optimal regimes (unconstrained and constrained) is studied. The formulation is made for a general aircraft performance model (general drag polar and general specific fuel consumption model), and is particularized for simpler models in order to establish the precise range of validity of some published results. The effects of wind on the optimal control laws and on the maximum range are studied. The accuracy of the incompressible approximation is also studied. Results are presented for a model of a typical twin-engine, wide-body, transport aircraft.

1. Introduction

The analysis of optimal cruise performance is important in long-range high-subsonic transport aircraft (see, for instance, Torenbeek's work [17,18]). When one considers Maximum Range Cruise (MRC), the problem is to maximize range for a given cruise fuel load. The MRC problem has been addressed by Miller [12] and Torenbeek [18], but in their analysis wind effects are ignored. In this paper a general analysis of this problem is presented, including wind effects. In this analysis the optimal control laws (that is, optimal altitude and speed laws) that maximize range are analyzed. Constrained regimes (constant-Mach or constant-altitude flights) are also considered; the corresponding optimal control law (optimal altitude or optimal speed law, respectively) is analyzed. The maximum range in the optimal regimes (unconstrained and constrained) is studied.

The classical problem of constant-altitude and constant-Mach cruise (studied thoroughly by Miele [11], Vinh [19], and more recently by Cavcar and Cavcar [3] and Cavcar [4], among others) is not considered in this paper; cyclic cruise, studied by Speyer [16] and Sachs and Christodoulou [15] among others, is not considered either.

In the previous references different aircraft performance models (APM) are considered. The most general case is that used by Torenbeek, namely, general drag polar and general specific fuel consumption model; this model is used in this paper.

Wind effects are considered by Hale and Steiger [5,6] and Asselin [1], but only for constant winds and using an oversimplified APM, namely, parabolic drag polar of constant coefficients and constant specific fuel consumption (model that is not appropriate for high-subsonic transport aircraft). Roskam and Lan [14] define a procedure to take into account wind effects in the calculation of range. The analysis of wind effects presented in this paper is made for a general APM (general drag polar and specific fuel consumption model) and takes into account the variation of wind speed with altitude. The only restriction is that crosswinds are ignored. The effects of wind on the optimal control laws and on the maximum range are studied.

The general formulation developed in this paper is also particularized for simpler APMs, and for constant winds. As a consequence of this analysis, some results obtained in the literature under some conditions are shown to be valid in more general cases; hence, the precise range of validity of some known results is established. The case of an incompressible drag polar is also considered; the accuracy of this approximation is studied.

Results are presented for an APM that models a typical twin-engine, wide-body, transport aircraft, including the optimal control laws, the maximum range, and the wind effects on both.

2. Formulation

2.1. Equations of motion

In civil air transport the cruise flight is quasi steady, thus in the analysis of aircraft cruise regimes the following equations are commonly used (see Miele [11]):

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1270-9638/$ – see front matter
doi:10.1016/j.ast.2009.11.005

$$T = D$$
$$L = W$$
$$\frac{\mathrm{d}r}{\mathrm{d}t} = V_g$$
$$\frac{1}{g}\frac{\mathrm{d}W}{\mathrm{d}t} = -cT \qquad (1)$$

The first two equations establish the equilibrium of forces, between thrust (T) and aerodynamic drag (D), and between lift (L) and aircraft weight (W); the 3rd equation is the kinematic equation, where r is the distance flown by the aircraft and V_g is the aircraft ground speed, which can be expressed as $\vec{V_g} = \vec{V} + \vec{w}$, where $\vec{V}$ is the aerodynamic velocity and $\vec{w}$ is the wind velocity (relative to the ground); and the 4th equation is the mass equation, where c is the specific fuel consumption (defined as mass of fuel consumed per unit thrust and unit time). Time (t) is the independent variable.

For the atmosphere, the ISA (International Standard Atmosphere) model is considered, which defines the density (ρ), pressure (p) and temperature (Θ) as functions of altitude (h); in particular, at sea level, $\rho_0 = 1.225$ kg/m^3, $p_0 = 101.326$ kN/m^2 and $\Theta_0 = 288.15$ K. Also, a constant-gravity model is adopted, defined by $g = 9.80665$ m/s^2.

The winds considered in this paper have the following properties: they are contained in the plane of flight (this means that crosswinds are not considered), have the same direction as $\vec{V}$, and can vary with altitude. Hence, one can write

$$V_g = V + w(\delta) \qquad (2)$$

where $\delta = \frac{p}{p_0}$ is the pressure ratio (which is a function of altitude). Thus, positive values of w correspond to tailwinds and negative values to headwinds.

2.2. Aerodynamic and propulsion models

The aerodynamic model provides the drag polar

$$C_D = C_D(M, C_L) \qquad (3)$$

that gives the drag coefficient as a function of Mach number, M, and lift coefficient, C_L. This is the general drag polar commonly used for this type of problems, see for instance Refs. [11,18].

In terms of the lift and drag coefficients, the lift and aerodynamic drag can be written as

$$L = q_0 \delta M^2 C_L$$
$$D = q_0 \delta M^2 C_D(M, C_L) \qquad (4)$$

where $q_0 = \frac{1}{2}\gamma p_0 S$, $\gamma = 1.4$ (ratio of specific heats) and S is the reference wing area. Using the 2nd equation of motion (1) one has for the lift coefficient the following expression

$$C_L = \frac{W}{q_0 \delta M^2} \qquad (5)$$

The propulsion model provides the specific fuel consumption. The following general model is considered

$$c = \frac{a_0\sqrt{\theta}}{L_H} C_C(M, C_T) \qquad (6)$$

where a_0 is the speed of sound at sea level, $\theta = \frac{\Theta}{\Theta_0}$ is the temperature ratio, L_H is the fuel latent heat, and C_C is the specific fuel consumption coefficient, which in general is a function of the Mach number and the thrust coefficient, C_T, defined as $C_T = \frac{T}{W_{TO}\delta}$, where W_{TO} is the reference take-off weight. Using the 1st equation of motion (1) and the definition of C_D (Eq. (4)) one has

$$C_T = \frac{q_0}{W_{TO}} M^2 C_D(M, C_L) \qquad (7)$$

In the applications found in the literature, the dependence of C_C with C_T is neglected, since in practice is very week [11]. On the contrary, the dependence of C_C with M is important for high bypass ratio turbofans [18], and must be taken into account. The fuel consumption associated to running all auxiliary equipment is not considered in the model.

In this paper, the general formulation is made for the general drag polar (Eq. (3)) and the general specific fuel consumption model (Eq. (6)). In the applications, results are presented for the parabolic drag polar of Cavcar and Cavcar [3] (with linear term and Mach-dependent coefficients) and the specific fuel consumption model proposed by Mattingly [10] and approximately depicted by Miele [11] (with the dependence of C_C with C_T neglected, and a linear model for $C_C(M)$). These models are described in more detail in Appendix A.

2.3. Range

The flight range is obtained integrating the kinematic equation (1), and can be transformed into a weight integration using the mass equation (1)

$$R = \int_{t_i}^{t_f} V_g \,\mathrm{d}t = -\frac{1}{g}\int_{W_i}^{W_f} \frac{V_g}{cT}\,\mathrm{d}W = \frac{1}{g}\int_{W_f}^{W_i} \frac{a_0\sqrt{\theta}M + w}{cD}\,\mathrm{d}W \qquad (8)$$

where W_i and W_f are the initial and final aircraft weights (the difference $W_F = W_i - W_f$ is the cruise fuel), and where Eq. (2), the definition of Mach number and the 2nd equation of motion (1) have been taken into account.

From Eqs. (4) and (5) one has that the functional dependence for the aerodynamic drag is $D = D(M, \delta, W)$, hence, the integrand is a function of M, δ, and W, so that one can write

$$R = \int_{W_f}^{W_i} S_R(M, \delta, W)\,\mathrm{d}W \qquad (9)$$

where the specific range S_R is given by

$$S_R = \frac{a_0\sqrt{\theta}M + w}{gcD} \qquad (10)$$

Once the control laws $M(W)$ and $\delta(W)$ are set, the range R can be calculated. These control laws represent the variation of Mach number and altitude with aircraft weight along the cruise flight.

3. Optimal control laws

The objective of the Maximum Range Cruise (MRC) is to maximize the flight range for a given cruise fuel load. In this section the optimal control laws that lead to range maximization are obtained. Since maximum range is obtained by maximizing at each weight the specific range (see Miele [11]), the optimal control laws follow from the partial derivatives of S_R (Eq. (10)) with respect to the control parameters M and δ, namely,

$$\frac{\partial S_R}{\partial M} = 0$$
$$\frac{\partial S_R}{\partial \delta} = 0 \qquad (11)$$

Hence, one obtains the following two optimal equations