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A systematic single-range controller synthesis procedure for nonlinear and multivariable liquid propellant engines

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Abstract

A new systematic single-range controller synthesis procedure for use with nonlinear, multivariable, and time-varying liquid propellant engines is developed. The developed procedure is based on one describing function model of the nonlinear plant coupled with two different linear algebraic controller design procedures; one of the algebraic procedures is to achieve decoupling, and the second one is to achieve command following. The developed procedure is demonstrated by solving an example problem comparing the results using a H_∞ controller design procedure.

Keywords: Describing functions; Factorization theory; Design methodologies; Fourier integrals; Propulsion control; H_∞ control

1. Introduction

In general, a typical liquid propellant engine may be classified as a nonlinear, multivariable, deterministic, and time-varying system. Controller design methods for such processes are limited [1–16]. Some of the popular techniques are: geometric transformation [2], Liaponouv and relay structure [3–6], quantitative feedback theory (QFT) [7], optimal [8], adaptive [9], and describing functions [10–16]. The geometric and relay structure approaches have made major strides in solving difficult nonlinear control problems. The QFT approach has shown to be effective in solving nonlinear control problems of a general nature. Adaptive techniques, using update laws based on a Liaponouv analysis, as well as optimal approaches, based on a Fourier approximation, have also been developed. Note that use of controller design based upon either of these approaches is usually justified if the classical control theory is not applicable. It should also be kept in mind that adaptive or optimal control laws are usually very difficult to implement. Traditionally, ordinary describing function (DF) techniques had primarily been used for system analysis (e.g. limit cycle prediction). In recent years, systematic design approaches, based on the describing function techniques, have enjoyed considerable success in achieving “robust” feedback systems that directly take into account the plant sensitivity issues. In this research, a describing function approach and two different algebraic approaches are used for design of a reliable multivariable and nonlinear control system; results are also compared with an alternative approach involving application of a H_∞ control theory [17,18]. The primary reason for using a describing function approach is that engine model is of the form given by the following state variable equations.

$$\dot{x} = f(x, u, t), \tag{1}$$

$$y = g(x, u, t), \tag{2}$$

where x is the vector of state variables, y is the vector of outputs, u is the vector of inputs, t is the time variable, and f, g are dummy nonlinear functions. Describing function approach is inherently capable of handling such nonlinear models. Of course, QFT approach is also capable of dealing with such models; however, the application of the QFT approach is left for future research.

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The problem statement follows. Given a computer model of a multivariable and nonlinear liquid propellant engine, how does one systematically design a control system for that liquid propellant engine? In this work an answer is provided. The primary contributions of this work are in two fold – (1) development of a new single-range controller synthesis procedure for use with nonlinear and multivariable liquid propellant engines, and (2) application of the presented design procedure and the associated software to a specific non-autonomous, multivariable, and nonlinear liquid propellant engine. Reference [11] is the key literature that this work is based upon; in that work the idea for design of single-range controllers is proposed, and in this work, a new single-range controller design procedure for multivariable and nonlinear systems is developed.

2. Description of controller synthesis procedure

The developed controller synthesis procedure is composed of 4 steps. Those steps are – (1) characterizing the input/output behavior of the system at one specific operating regime of interest, (2) identification of a linear model whose dynamic and static behavior mimics that of the previous step, (3) first use the factorization approach presented in [19], and design the main diagonal terms of the controller; then, design the off diagonal terms of the controller by using the algebraic approach given in [20]; also for comparison purposes, design a H_∞ controller, and (4) verify design.

Step 1: A Fourier based approach is used to obtain pseudo frequency response data known as SIDF models [16]. In short, these models are obtained by first exciting the multivariable nonlinear system by a sinusoid of the following form:

$$u_p(t) = u_{0,p} + a_p \cos(\omega_p t + \theta_p), \quad p = 1, 2, \ldots, m, \tag{3}$$

where u_p is the input, p is the input channel index, $u_{0,p}$ is the DC component of the input signal, a_p is the amplitude of excitation, ω_p is the frequency of excitation, and θ_p is the phase shift.

It is emphasized that the frequencies of excitation cannot be identical, because one would not be able to determine the separate effects of each input on each output. For this reason, various excitation frequencies must be related rationally, e.g., $\frac{\omega_1}{\omega_2} = \frac{1}{2}$ for the two-input case. The range of frequency is normally dictated by the physics of the problem; however, the user must make sure that he has considered enough low and high frequencies in order to completely characterize the low and high frequency behavior. The numerical values for phase shift may be selected by sweeping the angles from 0 to 360 degrees avoiding those angles that could make the presented describing function analysis faulty (e.g., $\theta = 90°$; in this case the nonlinear system would be restricted to be odd). Note that phase shift signifies the simultaneous interaction of nonlinear effects and couplings among the control loops. Therefore, for the single-range controller design, one may initially set the phase shifts equal to zero as recommended in [13,21].

Then, the equations of motion are numerically integrated to obtain the outputs y_q where q is the output channel index. When the outputs are at steady state, then the developed multivariable Fourier integrals are computed as follows:

$$I_{q,p}^{h,k} = \int_{(k-1)T}^{kT} y_q(t) \exp\bigl[-jh(\omega_p t + \theta_p)\bigr]\,\mathrm{d}t, \tag{4}$$

where k is the period index, h is the index for the considered harmonic, T is the overall period, and the remaining variables are defined as before. For a two-input two-output system, $T = \frac{2\pi}{|\omega_1 - \omega_2|}$. Finally, the multivariable SIDF models at discrete frequencies are denoted by $G_{q,p}^{1,k}(j\omega_q; u_0, a, \theta)$, and they are given by the following relation:

$$G_{q,p}^{1,k}(j\omega_q; u_0, a, \theta) = \frac{2}{a_p T} I_{q,p}^{1,k}. \tag{5}$$

In order to study the higher harmonic effects, one may set $h = 2, 3, \ldots$. It should be noted that Fourier integrals are said to have converged, when the following conditions are satisfied:

$$\left| \frac{M_{q,p}^k - M_{q,p}^{k-1}}{M_{q,p}^k} \right| < \varepsilon_M, \tag{6}$$

$$\left| \phi_{q,p}^k - \phi_{q,p}^{k-1} \right| < \varepsilon_\phi, \tag{7}$$

where $M_{q,p}^k$ is the magnitude of $G_{q,p}^{1,k}$ and $\phi_{q,p}^k$ is the phase of $G_{q,p}^{1,k}$. It should be noted that above approach is limited to stable systems. The listing of the software for generation of pseudo multivariable frequency response data is given in [22]. In this work, only one describing function model is assumed to adequately represent the dynamic behavior of nonlinear process.

The time-varying nature of the nonlinear plant is taken into account when simulating the plant and obtaining the corresponding output signals (i.e., the time-varying effects are imbedded in the output signals). Then, the output signals that appear in Eq. (4) are used to evaluate the Fourier integrals. If the effects of time-variance cause the Fourier integrals not to converge, then the approach may not be applicable at this time. As is shown in Section 3 of the paper, this is not of a major concern for the class of liquid-propellant engines under consideration.

Step 2: This step requires identification of a linear model whose frequency response data matches that of the previous step nominal SIDF model. Since, the describing function models are representation of nonlinear systems, the standard relation between the two components of the frequency response data that exist for linear systems, does not hold for DF models. Therefore, care must be taken when fitting the pseudo frequency response data in the sense that the user may wish to weigh data at some frequencies more than those of other frequencies (e.g., weighing the data more near the cross-over more than those at high frequencies) [23]. The outcome of this step is a linear model described in terms of a transfer function.

Step 3: The decoupling procedure is demonstrated for a typical 2×2 process. With reference to Fig. 1, the following relations hold:

$$y_i = e_i(g_{ii}c_{ii} + g_{ij}c_{ji}) + e_j(g_{ii}c_{ij} + g_{ij}c_{jj}) \quad \{i = 1, 2;\ j = 1, 2;\ i \neq j\}. \tag{8}$$