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Overview of some optimal control methods adapted to expendable and reusable launch vehicle trajectories

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Abstract

Launcher trajectory optimization is a complex task, especially when considering the specific problems arising in the study of reusable launch vehicles. Part of the difficulty comes from the different characteristics of the trajectory arcs which make up the vehicle's mission (constraints and controls may not be the same). Another difficulty is the necessity, in some cases, of a global optimization between ascent and re-entry phases (branching optimisation). Finally, optimization tools devoted to this task should be polyvalent and robust, as the studies of reusable launch vehicles usually cover many different concepts, and also many different trajectory cases (such as abort scenarios). The purpose of this paper is to present different approaches used in France by CNES and ONERA to solve optimal control problems in the context of launcher trajectory optimization. These approaches, which are powerful implementations of classical optimization methods, were designed to cover the needs for both expendable and reusable launchers trajectory calculation. The first optimization tool presented is OPTAX, which uses an indirect shooting method. The second and third tools presented are CNES's ORAGE and ONERA's FLOP/OLGA, which use two different variants of the gradient method. The paper describes the equations and methodology behind these tools, and also presents their advantages and drawbacks.

Keywords: Optimization; Launcher; Trajectory

Introduction

Trajectory optimization is an unavoidable activity for space transportation. Since the beginning of the space exploration a very large variety of methods has been developed following the evolution of the vehicles (missiles, rockets, shuttle and re-entry corps, etc.) and the missions (cf. [2]). Partially or fully reusable launch vehicles are being studied in preparation of the future of launch systems. The increased complexity of such vehicle, from architecture and technology point of view, is also visible from trajectory optimization point of view. Technically speaking, the vehicle's mission is made of multiple phases, often organized in a branching way (case of re-entry for multiple stages), and combining different dynamics and controls.

Historically, many tools have been developed in Europe to solve one or several problems in a row. The DIAMANT, EUROPA and ARIANE's heritage have brought us competitive and robust programs for conventional expendable launchers, but these tools are sometimes not well adapted to reusable launchers. The consequence is a multitude of tools and methods.

In this paper, we propose an overview of the most common tools used today at CNES and ONERA.

The first one is OPTAX, which uses an indirect shooting method. The second and third tools presented are CNES's ORAGE and ONERA's FLOP/OLGA, which use two different variants of the gradient method.

1. Indirect shooting method: OPTAX: Optimization of Ariane's trajectories (CNES)

OPTAX is the CNES main optimization tool for Ariane and all expendable launchers' ascent trajectories. It can also be used

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Nomenclature

t	time, between t_0 (initial) and t_f (final)
$X(t)$	state vector
$R(t)$	position vector of the launcher
$V(t)$	velocity vector of the launcher
a	parameter vector
$u(t)$	control vector (unitary) of the problem
$\lambda(t)$	adjoint vector of the state vector
ν	Lagrange multipliers
J	performance index (criterion)
ψ	final constraints vector
γ	intermediate constraints vector

Acronyms

CV	Calculus of Variation
DAE	Differential Algebraic Equations
FLOP	Future Launcher Optimization Program
MDO	Multi-Disciplinary Optimization
NLP	Non-Linear Programming
OC	Optimal Control
OPERA	Optimisation de PERformances Ariane
ORAGE	Atmospheric Re-entry Optimization using Extended Gradient Method
IRK	Implicit Runge–Kutta
RK	Runge–Kutta
RLV	Reusable-Launch Vehicle
TPBVP	Two-Point Boundary Value Problem

for reusable launchers within the restriction of an atmospheric phase having a constant or tabulated angle of attack.

OPTAX is based on a direct application of the Maximum Principle of Pontryagin. Practically, this principle is applied only for trajectory arcs outside the atmosphere (because we can easily obtain an explicit expression of the control). The atmospheric arcs are optimized parametrically.

1.1. Terms of the problem

We consider a general case with initial conditions, p parameters, q intermediate constraints, r final constraints and a free final time. The initial time here is t_{opt}, date of beginning of the optimal control (it is typically the time when aerodynamic forces become negligible).

The problem is to minimize a performance index:

$$J = \phi\bigl(X(t_f), t_f\bigr). \tag{1.1}$$

The objective function here has a Mayer form due to the fact that we rarely use an integral criterion for the ascent phase.

The state vector $X(t)$ is composed of the position and velocity vector $x(t) = (R(t)^{\mathrm{T}}, V(t)^{\mathrm{T}})^{\mathrm{T}}$, and the parameters a_k (payload mass, coast phase duration, tilting velocity, etc.). The control $u(t)$ describes the direction of the thrust vector, which is collinear to the vehicle's axis.

The minimization is subject to the following conditions:

$$\dot{X}(t) = f\bigl(X(t), u(t), t\bigr) \quad \text{dynamics}, \tag{1.2}$$

$$\varphi\bigl(x(t_{\mathrm{opt}})\bigr) = 0 \quad \text{initial constraint}, \tag{1.3}$$

$$\psi_i\bigl(X(t_f)\bigr) = 0 \quad r \text{ final constraints}, \tag{1.4}$$

$$\gamma_j\bigl(X(t_m)\bigr) = 0 \quad q \text{ intermediate constraints}. \tag{1.5}$$

Note that a_0 are the parameters that act during the non-optimal control phase before t_{opt} (atmospheric phase in general). The state vector at $x(t_{\mathrm{opt}})$ is completely defined by a_0.

The intermediate constraints can occur at optimized times $t_m \in [t_{\mathrm{opt}}, t_f]$.

The dynamic equations are:

$$\begin{cases} \dot{R} = V, \\ \dot{V} = \sum F/M(t), \\ \dot{a} = 0. \end{cases} \tag{1.6}$$

F represents the thrust and weight forces (aerodynamic forces are neglected after t_{opt}) and M the mass (function of time). Parameters a_k are fixed in the time.

1.2. Optimality conditions

In a similar manner of the Lagrangian functions, we construct an augmented performance index (cf. [6]):

$$\hat{J} = \bigl[\nu_0^{\mathrm{T}}\varphi\bigr]_{t_{\mathrm{opt}}} + \left[\nu_f\phi + \sum_1^r \nu^{\mathrm{T}}\psi\right]_{t_f} - \int_{t_{\mathrm{opt}}}^{t_f} \lambda^{\mathrm{T}}\bigl[\dot{X} - f(X, u, t)\bigr]\,\mathrm{d}t. \tag{1.7}$$

We have omitted the term of intermediate constraints (γ_m) for simplification purpose. We introduce the Lagrange multipliers ν for the final constraints and the adjoint vector $\lambda(t)$ of the state vector for the dynamic constraint.

$$\lambda(t) = \begin{pmatrix} \lambda_x(t) \\ \lambda_a(t) \end{pmatrix} = \begin{pmatrix} \lambda_R(t) \\ \lambda_V(t) \\ \lambda_a(t) \end{pmatrix}. \tag{1.8}$$

The necessary optimality conditions are given by setting: $\delta\hat{J} = 0$.

$$\delta\hat{J} = \int_{t_{\mathrm{opt}}}^{t_f} \left[\left(\lambda^{\mathrm{T}} \cdot \frac{\partial f}{\partial X} + \dot{\lambda}^{\mathrm{T}}\right)\delta X + \left(\lambda^{\mathrm{T}} \cdot \frac{\partial f}{\partial u}\right)\delta u\right]\mathrm{d}t + \left[\nu_0^{\mathrm{T}}\frac{\partial\varphi}{\partial X} + \lambda^{\mathrm{T}}\right]_{t_{\mathrm{opt}}} \mathrm{d}X_{\mathrm{opt}} + \left[\nu_f\frac{\partial\phi}{\partial X} + \nu^{\mathrm{T}}\frac{\partial\psi}{\partial X} - \lambda^{\mathrm{T}}\right]_{t_f} \mathrm{d}X_f$$