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The rotor theories by Professor Joukowsky: Vortex theories

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ABSTRACT

This is the second of two articles with the main, and largely self-explanatory, title “Rotor theories by Professor Joukowsky”. This article considers rotors with finite number of blades and is subtitled “Vortex theories”. The first article with subtitle “Momentum theories”, assessed the starring role of Joukowsky in aerodynamics in the historical context of rotor theory. The main focus in both articles is on wind turbine rotors, but much of the basic theory applies to propellers and helicopters as well. Thus this second article concentrates on the so-called blade element theory, the Kutta–Joukowsky theorem, and the development of the rotor vortex theory of Joukowsky. This article is to a large extent based on our own work, which constitutes the first successful completion and further development of Joukowsky's work by deriving the first analytical solution of his rotor. This rotor has a finite number of blades and will be compared with the rotor analysis of Betz and of others of the German school of aerodynamics.

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1. Introduction

This is the second of two articles with the main, and largely self-explanatory, title “Rotor theories by Professor Joukowsky”. This article considers rotors with finite number of blades and is subtitled “Vortex theories”. The first article with subtitle “Momentum theories” [1], assessed the starring role of Joukowsky in aerodynamics in the historical context of rotor theory. The main focus in both articles is on wind turbine rotors, but much of the basic theory applies to propellers and helicopters as well. Froude's momentum theory for an actuator disc was the first elementary one-dimensional model of a rotor which was sufficiently accurate to describe correctly the averaged and simplified structure of the flow, and establish the Betz–Joukowsky limit for the optimum performance of wind turbines. We have ascertained the role of Joukowsky in the derivation of this important limit on wind power conversion efficiency. Joukowsky's general momentum theory for actuator discs became the next stage in the development of rotor aerodynamics [1]. This theory followed from his understanding of the physical principles of the rotor operation based on vortex theory of screw propellers with constant circulation along the blade. In Ref. [1] we have analyzed the difficulties encountered when applying the general rotor momentum theory proposed by Joukowsky about a century ago on wind turbines. Undoubtedly, the general momentum theory of the rotor, as well as the simpler theory of Rankine–Froude, is still far from perfect due to simplifying assumptions and conditions.

The simple momentum theories described in Ref. [1] were not sufficient to design propeller blades, which was the most important rotor configuration of the time. Therefore, in parallel, a specific design method was developed, which was based on dividing a blade into a number of span-wise sections and using vortex theory to determine the induced velocities. Thus this second article concentrates on the so-called blade element theory, the Kutta–Joukowsky theorem, and the development of the rotor vortex theory of Joukowsky. The article is to a large extent based on our own work, which constitutes the first successful completion and further development of Joukowsky's work by deriving the first analytical solution of the NEJ rotor. The acronym NEJ comes from the initial letters of Joukowsky's name (Nikolay Egorovich Joukowsky[1]). This rotor has a finite number of blades and will be compared with the rotor analysis of Betz and of others of the German school.

Section 2 presents new historical facts regarding the Kutta–Joukowsky (KJ) theorem, as well as a review of the blade element momentum (BEM) method and the development of rotor vortex theories for estimating helical vortex structures, and Joukowsky's role in this development. Section 3 presents the most important steps in the development of the BEM theory, concluded with a derivation of the KJ theorem for a cascade of blade elements. The use of the KJ theorem to lifting line theory of rotors with finite number of blades is continued in Section 4, where the final results are presented. Furthermore, the section provides a comparison and a critical review of different erroneous vortex theories of rotors.

2. Blade element momentum and vortex theories: retrospect

2.1. The history of the Kutta–Joukowsky (KJ) theorem

The history of Kutta and Joukowsky (Fig. 1) and their famous equation or theorem (KJ) is, even among experts, almost unknown.

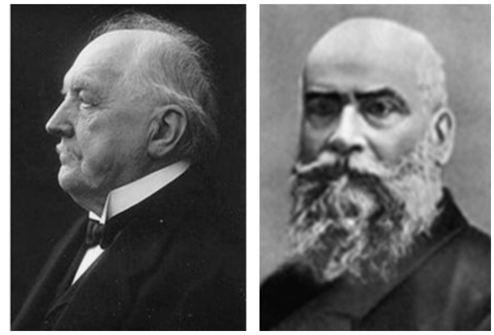

Fig. 1. Originators of the Kutta condition and the KJ theorem: M. Kutta and N.E. Joukowsky.

For example, Anderson [2] (page 391) correctly credits Joukowsky [3] for deriving the KJ equation in 1906, but claims at the same time that Joukowsky was unaware of Kutta's work [4] (published in 1902: English translation in Ackroyd et al. [5]). This may not be strictly true, although there is no reference to Kutta in the 1906 paper. However, this may not be relevant, as Joukowsky without doubt was aware of the Kutta condition. Furthermore, Kutta did not derive the KJ equation. According to Panton [6] (page 427), the KJ equation was named “after the two people who discovered it independently”. This may be the result of a secondary referencing, as Lamb [7] (page 681) surprisingly also attributes the equation to both.[2]

Kutta [4] derived an equation for the lift for a thin, circular arc airfoil in an inviscid flow obeying the Kutta condition, which states that the flow leaves the airfoil smoothly at the trailing edge. He did not name the condition and he did not mention circulation and its relationship to the lift force. In 1910 Joukowsky [8] reviewed Kutta's work, re-derived his equation for lift, introduced the circulation and denoted the trailing edge conditions the Kutta condition, probably for the first time.[3] He then derived the Kutta–Joukowsky (KJ) theorem relating lift to the circulation around a body immersed in a two-dimensional flow of an otherwise inviscid fluid. This paper is one of the masterpieces of early aerodynamics research.

In 1906, between these two papers, came Joukowsky [3], whose title can be translated as “On Bound Vortices”, although it appears in Ackroyd et al. [5] as “On Annexed Vortices”. He associated the body with the closed streamlines around a vortex of specified strength. He chose (in effect) a large circular control volume (CV) of radius R centered on the body and used the condition that the velocity potential of the flow perturbed by a body decays as R^{-2} for sufficiently large R, so that there is no efflux of momentum from the CV. Thus the force due to pressure at R, determined using Bernoulli's equation, must be equal and opposite to that on the body. From this he derived the general form of what is now called the KJ equation, which is derived in a simpler manner below as Eq. (35). In 1910 Kutta [9] (English translation in Ackroyd et al. [5]) referred to Joukowsky [3] and extended the analysis of the circular arc airfoils. He also noted that his thesis [4] contained a much more detailed analysis including the KJ theorem. That thesis has

[1] Alternatively, his name is spelled Zhukovskii, Joukowski, Joukovskii or Žukovskij etc.

[2] Ref. [7] is the last, 1945 edition. There is no reference to Kutta in the 1906 edition. The comments on the KJ equation cited here were also in the 1916 edition which introduced them.

[3] In this connection it is interesting that the Kutta condition has been known as the Joukowsky–Chaplygin condition in Russia (Prof. S.A. Chaplygin was a colleague of Joukowsky). Consequently, in order to honor all initiators, this result should be re-named the “Kutta–Joukowsky–Chaplygin condition”. The well-established and convenient name Kutta condition should be considered as an easy abbreviation of this full name.