Propulsion and Power Research 2016;5(2):118–122

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Propulsion and Power Research

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ORIGINAL ARTICLE

Mixed convection boundary layer flow past a vertical cone embedded in a porous medium subjected to a convective boundary condition

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Received 10 March 2015; accepted 29 May 2015
Available online 17 May 2016

KEYWORDS
Mixed convection;
Porous medium;
Convective boundary condition;
Dual solutions;
Fluid mechanics

Abstract In the present analysis, we study the steady mixed convection boundary layer flow past a vertical cone embedded in a porous medium subjected to a convective boundary condition. The governing partial differential equations are reduced to the coupled nonlinear ordinary differential equations using a similarity transformation before being solved numerically by a shooting method. Both assisting and opposing flows are considered. The influence of the convective heat transfer parameter is analysed and discussed through graphs. Dual solutions are found to exist for the case of opposing flow.

1. Introduction

Mixed convection flow plays an important role in many engineering processes. The combination of forced and free convection flow, arise in many transport processes such as heat exchangers, solar collectors, nuclear reactors, electronic equipment etc. The mixed convection flow is important when the buoyancy force due to the temperature difference between the free stream and the solid surface become large, which in turn significantly affect the flow and the thermal fields. Ramachandran et al. [1] investigated the problem of mixed convection in two dimensional stagnation flow adjacent to a vertical surface by considering both cases of an arbitrary wall temperature and arbitrary surface heat flux variations. They concluded that a reverse flow region

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Peer review under responsibility of National Laboratory for Aeronautics and Astronautics, China.

http://dx.doi.org/10.1016/j.jppr.2016.04.005

developed in the buoyancy opposing flow region, and dual solutions are found to exist in that flow regime for a certain range of the buoyancy parameter. Devi et al. [2] extended this work to the unsteady case where the unsteadiness in the flow and temperature fields is caused by the time-dependent of the free stream velocity. The similarity solutions for mixed convection boundary-layer flow when the wall heat flux is prescribed was analysed in details by Merkin and Mahmood [3].

Merkin [4] considered the mixed convection boundary layer flow about a vertical flat impermeable surface embedded in a saturated porous medium. Ishak et al. [5] investigated the mixed convection flow near a stagnation point on a vertical porous plate. They found that dual solutions exist for the assisting flow whereas in the opposing region, the solution is unique. Bachok et al. [6] discussed the steady mixed convection boundary-layer flow near the stagnation point on a vertical flat plate embedded in a fluid-saturated porous medium characterised by an anisotropic permeability. They concluded that the effect of anisotropy is to increase the range of the modified mixed convection parameter for which the solution exists.

Cheng et al. [7] analysed the natural convection of a Darcian fluid about an inverted heated cone embedded in a porous medium of infinite extent. They realized that the solution behaves like that of an inclined plate at a location near the leading edge. The problem of heat transfer by mixed convection flow over a vertical cone embedded in a saturated porous medium was investigated by Yih [8]. Nonsimilar solutions were obtained for the cases of variable wall temperature and variable heat flux. Yih [9] studied the effect of radiation on mixed convection flow of an optically dense viscous fluid adjacent to an isothermal cone embedded in a saturated porous medium.

Although numerous studies have considered different variations of temperature and heat flux at the plate, not many study appeared to have considered a convective boundary condition at the plate. However, Aziz [10] considered a convective surface boundary condition in the classical problem of hydrodynamic and thermal boundary layer over a flat plate. Magyari [11] recovered the exact solution of the temperature problem in a compact integral form. Since the numerical results reported in [10] for $Pr=0.1$ are not enough accurate, Ishak [12] extended the work of Aziz [10] by introducing the effects of suction and injection on the flat surface, besides giving an accurate numerical results for $Pr=0.1$.

The focus of the present paper is to investigate numerically the problem of the steady mixed convection boundary layer flow past a vertical cone embedded in a porous medium subjected to a convective boundary condition.

2. Mathematical formulation

Consider a steady mixed convection boundary layer flow past a vertical cone with the local radius r and half angle ϕ embedded in a fluid saturated porous medium. It is assumed that the surface of the cone is heated or cooled by flowing fluid at a constant temperature T_f with variable heat transfer coefficient $h_f(x)$, while the temperature of the ambient fluid is T_∞, where $T_f > T_\infty$ corresponds to a heated cone and $T_f < T_\infty$ corresponds to a cooled cone, respectively. It is also assumed that the uniform free stream velocity U_∞ is oriented in the upward direction. The physical model is shown in Figure 1, where the x axis is measured along the surface of the cone and y is the coordinate measured normal to it. The origin of the coordinate system is placed at the vertex of the cone. Under the assumption of Darcy law, the basic equations can be written in the Cartesian coordinates x and y as (see Nield and Bejan [13]),

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0 \tag{1}$$

$$\frac{\partial u}{\partial y} = \frac{gK\beta\cos\phi}{\nu}\frac{\partial T}{\partial y} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where $r(x) = x\sin\phi$, u and v are the velocity components along the x and y axes, respectively, T is the fluid temperature, K is the permeability of the porous medium, g is the acceleration due to gravity, α_m is the effective thermal diffusivity of the porous medium, β is the coefficient of thermal expansion and ν is the kinematic viscosity. Following Aziz [10], we assume that the boundary conditions of Eqs. (1)–(3) are

$$u = v = 0 \quad \text{at} \quad y = 0$$
$$u \to U_\infty, \quad T \to T_\infty \quad \text{as} \quad y \to \infty \tag{4a}$$

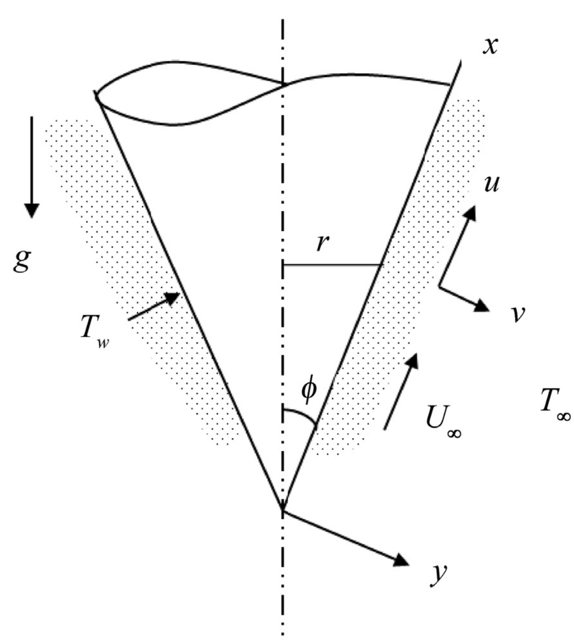

Figure 1 Physical model and coordinate system.