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ORIGINAL ARTICLE

Nonlinear dynamic analysis of a punctual charge in the electric field of a charged ring via modified frequency–amplitude formulation

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Abstract In this paper, two types of frequency amplitude formulation method are initially utilized to obtain frequency–amplitude relationship of nonlinear vibration of a punctual charge in the electric field of a charged ring. In order to obtain the nonlinear natural frequency of the considered system, Reng-Gui and Geng-Cai modified methods are implemented. A table is also prepared to provide a brief review of recent development of nonlinear differential equations. The correctness of the obtained results is compared with those obtained from harmonic balance method (HBM) and energy balance method (EBM). A numerical simulation is carried out to investigate the accuracy of the used methods. In accordance with it, the relative errors of the employed approaches are numerically and analytically found based on the exact numerical solutions. It is exposed that the exerted approaches are very reliable and applicable for solving the nonlinear differential equations.

1. Introduction

Nonlinear differential equations are the best way to show complicated behavior of the different phenomena in the real world [1]. Understanding and predicting of the physics of lots

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Table 1 Different types of nonlinear differential equations.

Nonlinear differential equations	Name of equation
$\ddot{x}+\dfrac{x}{x^2+1}=0$	Nonlinear oscillator with rational term [5]
$\ddot{x}+\dfrac{x^3}{x^2+1}=0$	Duffing-harmonic oscillator [5]
$\ddot{x}+\dfrac{x}{\sqrt{x^2+1}}=0$	Relativistic oscillator [5]
$\ddot{x}+\dfrac{1}{x}=0$	Plasma physics equations [5]
$\ddot{x}+x\lvert x\rvert^{\alpha-1}=0$	Nonlinear oscillator with fractional power [8]
$\ddot{x}+2\xi\dot{x}+\varepsilon\,\mathrm{sgn}(x)\lvert x\rvert^{\alpha}=F\cos\Omega t$	Forced vibrations of oscillators with a purely nonlinear power-form restoring force [9]
$\ddot{x}-\varepsilon(1-x^2)\dot{x}+x=\varepsilon\gamma D^{\alpha}(y-x)$ $\ddot{y}-\varepsilon(1-y^2)\dot{y}+y=\varepsilon\gamma D^{\alpha}(x-y)$	van der Pol oscillators coupled by fractional derivatives [10]
$\ddot{x}+(\delta+\varepsilon\cos t)x+cD^{\alpha}x=0$	Fractional Mathieu equation [11]
$u_t+uu_x+g\eta_x=0$ $\eta_t+(u(\eta+H))_x=0$	Wave propagation in a shallow media [12]
$(\overline{a}\cos^2\theta+\overline{b}\sin^2\theta)\ddot{\theta}+0.5(\overline{b}^2-\overline{a}^2)\dot{\theta}^2+g\overline{b}\sin\theta=0$	Nonlinear lateral sloshing in partially filled elliptical tankers [13]
$i\psi_t+\psi_{xx}+\gamma\lvert\psi\rvert^2\psi=0$	Schrödinger equation [14]
$\dfrac{dT}{dt}=q-\alpha T+\gamma T(1-\dfrac{T+1}{T_{\max}})-kVT$ $\dfrac{dI}{dt}=kVT-\beta I$ $\dfrac{dV}{dt}=N\beta I-\gamma V$	Model for HIV infection of CD4+ cells [15]
$\phi_{xt}+\phi_x\phi_{xy}+\frac{1}{2}\phi_{xx}\phi_y+\frac{1}{4}\phi_{xxxy}=0$	Bogoyavlenskii–Schieff equation [16]
$\dddot{x}=-\gamma\dot{x}-\alpha\dot{x}^3-\beta x^2\dot{x}-\delta x\ddot{x}-\varepsilon\dot{x}\ddot{x}^2$	Jerk equation [17]
$\dfrac{\partial^2u(x,t)}{\partial t^2}+\alpha\dfrac{\partial u(x,t)}{\partial t}+\beta u(x,t)=\dfrac{\partial^2u(x,t)}{\partial x^2}+f(x,t)$	Hyperbolic telegraph equation [18]
$\dfrac{dS}{dt}=(1-p)\pi N-\beta\dfrac{SI}{N}-\mu S,$ $\dfrac{dI}{dt}=\beta\frac{SI}{N}-(\gamma+\mu)I,$ $\dfrac{dR}{dt}=P\pi N+\gamma I-\pi R$	SIR epidemic model [19]
$\ddot{x}+\dfrac{x}{\sqrt[3]{1+x^2}}=0$	Punctual charge in the electric field of a charged ring [24–26]

of systems are very difficult owing to existence of nonlinearity. To have better view about our nature, solving of these equations are very effectual. Indeed, a closed form analytical solution for a nonlinear differential equation which describes a real phenomenon can provide a succinct and effective pack about behavior of a real system. So, a number of researchers devoted their time and efforts to find the most accurate way for solving of nonlinear problems. Perturbation methods [2], Homotopy Analysis method [3], Variational iteration method [4], Energy Balance method [5], Hamiltonian approach [6] and Frequency–Amplitude formulation [7] can be mentioned as powerful methods which have been presented and modified by several researchers. These powerful approaches have been employed to solve different kinds of nonlinear problems. Several examples of nonlinear differential equations are presented in Table 1. Aim of the present work is to obtain nonlinear frequency of the vibration of punctual charge in the electric field of a charged ring using two types of modified frequency amplitude formulation approaches. Frequency amplitude formulation approach has been recently proposed based on an ancient Chinese mathematical method [20]. Subsequently, this method has been modified and employed by several researchers to solve different types of nonlinear differential equations [21]. Geng-Cai and Reng-Gui new approaches can be referred as two of recent and effective modification of the abovementioned method [22,23]. The background of the FAF and brief review regarding the method has been provided in a review article by J.H. He [27]. FAF has been employed for analyzing the strongly nonlinear oscillators like a mass attached to a stretched wire by Akbarzadeh and Farshidianfar. They demonstrated that FAF is very powerful for studying dynamical behavior of the abovementioned system [28]. He's amplitude–frequency formulation is applied to study the periodic solutions of a strongly nonlinear system. This system corresponds to the motion of a mass attached to a stretched wire. The usefulness and effectiveness of the proposed technique is illustrated. The results are compared with exact solutions and those obtained by the harmonic balance. Approximate frequencies are valid in the entire range of vibration amplitudes. The agreement between the approximate and exact frequencies is demonstrated and discussed. In this study, these novel approaches are developed to find closed