Propulsion and Power Research 2014;3(1):41–47

http://ppr.buaa.edu.cn/

Propulsion and Power Research

www.sciencedirect.com

ORIGINAL ARTICLE

Approximate solution of the nonlinear heat transfer equation of a fin with the power-law temperature-dependent thermal conductivity and heat transfer coefficient

Sobhan Mosayebidorcheh[a,*], D.D. Ganji[b], Masoud Farzinpoor[c]

[a]Department of Mechanical Engineering, Islamic Azad University, Najafabad Branch, Isfahan, Iran
[b]Department of Mechanical Engineering, Babol University of Technology, Babol, Iran
[c]Department of Mechanical Engineering, Islamic Azad University, Khomeini-Shahr Branch, Isfahan, Iran

Received 23 October 2013; accepted 10 January 2014
Available online 18 March 2014

KEYWORDS

Differential transform method;
Nonlinear heat transfer equation;
Power-law temperature dependent;
Temperature distribution;
Fin efficiency

Abstract In this paper, differential transform method (DTM) is used to solve the nonlinear heat transfer equation of a fin with the power-law temperature-dependent both thermal conductivity and heat transfer coefficient. Using DTM, the differential equation and the related boundary conditions transformed into a recurrence set of equations and finally, the coefficients of power series are obtained based on the solution of this set of equations. DTM overcame on nonlinearity without using restrictive assumptions or linearization. Results are presented for the dimensionless temperature distribution and fin efficiency for different values of the problem parameters. DTM results are compared with special case of the problem that has an exact closed-form solution, and an excellent accuracy is observed.

1. Introduction

Fins are used to increase the heat transfer of heating systems such as, refrigeration, cooling of oil carrying pipe, cooling electric transformers, cooling of computer processor and air conditioning. A review about the extended surfaces and its industrial applications is presented by Kern and Krause [1]. Numerous researches have been done to investigate the heat transfer of the fins.

*Corresponding author: Tel.: +98 919 743 0343.
E-mail address: sobhanmosayebi@yahoo.com (Sobhan Mosayebidorcheh).

Peer review under responsibility of National Laboratory for Aeronautics and Astronautics, China.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.jppr.2014.01.005

Group classification of the differential equation of fin has been analyzed using symmetry analysis [2,3]. In another work, Pakdemirli and Sahin [4] investigated nonlinear equation of fin with general temperature-dependent thermal conductivity.

A simple state which the thermal conductivity and heat transfer coefficient are constant, the exact analytical solution is existent. But if a large temperature difference exists within a fin, heat transfer coefficient and thermal conductivity are not constant. Because of this, in general, thermal conductivity and heat transfer coefficient are functions of temperature.

It is clear that obtaining the exact solutions of these nonlinear problems are usually difficult. Because of this, researchers used the numerical techniques and semi-analytical methods such as the perturbation method (PM), homotopy perturbation method (HPM), variational iteration method (VIM), homotopy analysis method (HAM), decomposition method (DM) and differential transform method (DTM).

In this section, a short review about the related works is presented. This report specially includes the solution of the heat transfer equation of a fin with temperature-dependent thermal conductivity and/or temperature-dependent heat transfer coefficient using semi-analytical methods.

The oldest works in this subject have been done by Aziz and Hug [5] and Aziz and Benzies [6]. They solved the heat transfer equation of a convective fin with linear temperature-dependent thermal conductivity using perturbation method.

Heat transfer equation of a fin with linear temperature-dependent thermal conductivity and power-law temperature-dependent heat transfer coefficient has been considered by some researchers. Its differential equation and boundary conditions are in the following form [7]:

$$(1+\beta\theta)\frac{d^2\theta}{dx^2}-M^2\theta^{n+1}+\beta\left(\frac{d\theta}{dx}\right)^2=0,$$
$$BC's\rightarrow\begin{cases}\theta'(0)=0\\ \theta(1)=1\end{cases}. \quad (1)$$

This problem has been investigated using HAM [7], Taylor transformation and Adomian decomposition method (ADM) [8]. Khani et al. [7] says that solutions of HPM and ADM fail when M increases to a large number but HAM solution remains accurate. Peculiar case of Eq. (1) for $n=-1$ solved by Rajabi et al. using HPM [9]. Nonlinearity of the Eq. (1) reduces when the thermal conductivity is a constant ($\beta=0$ in Eq. (1)):

$$\frac{d^2\theta}{dx^2}-M^2\theta^{n+1}=0,$$
$$BC's\rightarrow\begin{cases}\theta'(0)=0\\ \theta(1)=1\end{cases}. \quad (2)$$

Lesnic and Heggs [10] and Chang [11] investigated the above equation with DM and ADM, respectively. Chowdhury et al. [12] solved the Eq. (2) using the HPM and HAM and a comparison between these results and ADM solution presented. Their study shows that ADM and HPM are the peculiar cases of the HAM concerning this problem. Eq. (2) for special example ($n=3$) indicates the radiation heat transfer of a fin to a free space. Solution of This case has been studied using PM, HPM [13] and VIM [14]. Perturbation results are much different with the exact solutions rather than HPM, because the small parameter in PM exists.

Another special case of the Eq. (1) occurs when the heat transfer coefficient is a constant and the thermal conductivity is a linear function of temperature ($n=0$ in Eq. (1)). This case has been solved with some semi analytical methods, such as PM [15], HPM [15,16], VIM [15,17], HAM [16,18], DTM [19] and ADM [20].

In this work, a nonlinear fin with the power-law temperature-dependent both thermal conductivity and heat transfer coefficient is considered. Then DTM is used to obtain an approximation solution of the problem. The results are validated for a special case that exact closed-form solution of the problem is existent in [21]. Accuracy check of the proposed method is presented by increasing the number of the Taylor series components. At the end, results of DTM demonstrated as the temperature distribution and fin efficiency for different values of the problem parameters.

2. Differential transform method

Zhou [22] introduced the concept of differential transform method to solve linear and nonlinear initial value problems. He used DTM to present the approximate solution for electrical circuit analysis. DTM is an iterative method to obtain the Taylor series of the solution. In recent years, the DTM has been used for solving a wide range of the differential equations, such as differential algebraic equations [23], nonlinear ordinary differential equations [24–27], partial differential equations [28–30], fractional differential equations [31,32], and integral equations [33].

The differential transform is defined as follows:

$$F(h)=\frac{1}{h!}\left[\frac{d^hf(t)}{dt^h}\right]_{t=t_0}. \quad (3)$$

Where, $f(t)$ is an arbitrary function and $F(h)$ is the transformed function. The inverse transformation is as follows

$$f(t)=\sum_{h=0}^{\infty}F(h)(t-t_0)^h. \quad (4)$$

Substituting Eq. (3) into the Eq. (4), we have

$$f(t)\approx\sum_{h=0}^{m}F(h)(t-t_0)^h. \quad (5)$$

The function $f(t)$ is usually expressed as a finite series and Eq. (5) can be rewritten as

$$f(t)\approx\sum_{h=0}^{m}F(h)(t-t_0)^h. \quad (6)$$