Propulsion and Power Research 2015;4(3):169–178

HOSTED BY

ELSEVIER

http://ppr.buaa.edu.cn/

Propulsion and Power Research

www.sciencedirect.com

A numerical study for off-centered stagnation flow towards a rotating disc

M. Heydari[a],*, G.B. Loghmani[a], M.M. Rashidi[b], S.M. Hosseini[c]

[a]*Department of Mathematics, Yazd University, P.O. Box 89195-741, Yazd, Iran*
[b]*Mechanical Engineering Department, Engineering Faculty of Bu-Ali Sina University, Hamedan, Iran*
[c]*Department of Mathematics, Khatam Center, Islamic Azad University, Yazd, Iran*

Received 24 March 2014; accepted 5 May 2015
Available online 1 October 2015

KEYWORDS

Off-centered stagnation flow;
Similarity transform;
Bernstein polynomials;
Function approximation;
Tau method

Abstract In this investigation, a semi-numerical method based on Bernstein polynomials for solving off-centered stagnation flow towards a rotating disc is introduced. This method expands the desired solutions in terms of a set of Bernstein polynomials over a closed interval and then makes use of the tau method to determine the expansion coefficients to construct approximate solutions. This method can satisfy boundary conditions at infinity. The properties of Bernstein polynomials are presented and are utilized to reduce the solution of governing nonlinear equations and their associated boundary conditions to the solution of algebraic equations. Graphical results are presented to investigate the influence of the rotation ratio α on the radial velocity, azimuthal velocity and the induced velocities. A comparative study with the previous results of viscous fluid flow in the literature is made.

1. Introduction

Many of the mathematical modeling, which appears in many areas of scientific fields such as fluid dynamics, plasma physics and solid state physics, can be modeled by nonlinear ordinary or partial differential equations. Such problems often require advanced numerical methods or powerful analytical methods to solve the governing equations. These known methods are for example, Runge-Kutta method [1], spectral methods [2,3], the δ-expansion method [4], the Adomian decomposition method [5], the variational iteration method [6,7], the homotopy perturbation method [8,9] and the homotopy analysis method [10,11].

---

*Corresponding author: Tel.: +989357954044.

E-mail address: m.heydari85@gmail.com (M. Heydari).

Peer review under responsibility of National Laboratory for Aeronautics and Astronautics, China.

http://dx.doi.org/10.1016/j.jppr.2015.07.004

The flow due to rotating disks is one of the classical problems of fluid mechanics which has many practical applications in manufacturing processes in industry, such as rotating machinery, crystal growth processes, cooling of silicon wafers and chemical vapor deposition processes etc. The pioneering study of fluid flow due to an infinite rotating disk has been carried out by von Karman [12]. He first formulated the problem and then reduced the governing partial differential equations to the ordinary differential equations by defining appropriate transformations. The stagnation flow problem was studied by Homann [13]. Hannah [14] introduced and discussed the combination of axisymmetric stagnation flow on a rotating disc. Tifford et al. [15] extended Hannah's work and combined it with torque. In the von Karman's rotating disc problem and Homann's stagnation flow problem an axisymmetric stagnation flow aligned with the axis of the rotating disc is considered. Since the flow is axisymmetric, the governing equations are greatly simplified.

Here, we have considered the off-centered stagnation flow toward a rotating disc. This problem was first studied by Wang [16] in 2008 where they implemented a similarity solution. Dinarvand [17] used the homotopy analysis method to solve off-centered stagnation flow toward a rotating disc. Nourbakhsh et al. [18] obtained an approximate analytical solution via homotopy analysis method with two auxiliary parameters. Rashidi et al. [19] introduced a combination of the differential transformation method and the Padé approximants for solving off-centered stagnation flow toward a rotating disc.

In this study, we are going to introduce and implement a new algorithm based on Bernstein polynomials [20] to find the approximate solution of the off-centered stagnation flow toward a rotating disc. Bernstein polynomials have many useful properties, such as, the positivity, the continuity, and unity partition of the basis set over the interval $[a, b]$. The Bernstein polynomials vanish except the first polynomial at $x = a$, which is equal to 1 and the last polynomial at $x = b$, which is also equal to 1 over the interval $[a, b]$. This provides greater flexibility in imposing boundary conditions at the end points of the interval.

The Bernstein polynomials are widely used for numerical solutions of differential, integral, and integro-differential equations which we point to some of them studied in recent years briefly. In Ref. [21], an algorithm for solving KdV equation using modified Bernstein polynomials is presented. These polynomials are applied to numerical solution of some classes of integral equations In Ref. [22]. Bhatti et al. [23] used the Bernstein polynomial basis to solve differential equations. Bhattacharya and Mandal [24] obtained numerical solutions of Volterra integral equations using the Bernstein polynomials. Chakrabarti and Martha [25] proposed an effective approach using the Bernstein polynomials to obtain approximate solutions of Fredholm integral equations of the second kind. Singh et al. [26] and Yousefi and Behroozifar [27] have proposed some operational matrices in different ways for solving differential equations. Authors of [28] applied the operational matrices of Bernstein polynomials for solving the parabolic equation subject to specification of the mass. Doha et al. [29,30] employed two attractive algorithms based on the derivatives and Integrals of Bernstein polynomials for solving high even-order differential equations. In Ref. [31] a numerical solution of the nonlinear age-structured population models by using the operational matrices of Bernstein polynomials is presented. Maleknejad et al. [32] proposed a Computational method based on Bernstein operational matrices for nonlinear Volterra-Fredholm-Hammerstein integral equations. Rostamy and Karimi [33] solved fractional heat- and wave-like equations using Bernstein polynomials. Some numerical integration methods based on Bernstein polynomials are introduced in Ref. [34]. In Ref. [35], a combination of homotopy analysis and tau Bernestein polynomial method is applied to solve singularly perturbed boundary value problems.

This paper is divided to the following sections. In Section 2, the flow analysis and mathematical formulation are presented. Section 3 describes the basic formulations of Bernstein polynomials required for our subsequent development. In Section 4, the approximate solution of the governing equations using Bernstein polynomials is presented. Section 5 contains the results and discussion. Finally, conclusions are made in Section 6.

2. Flow analysis and mathematical formulation

The description of the physical problem closely follows that of Wang [16]. Let u, v and w be the velocity components along x, y and z directions, respectively. A stagnation flow along the z axis impinging on a rotating disc whose axis is distance b from that axis is depicted in Figure 1.

The appropriate conditions at infinity on the potential stagnation flow are

$$u = ax, \quad v = ay, \quad w = -2az, \tag{1}$$

where a is the strength of the stagnation flow. The boundary conditions on the disc are

$$u = -\Omega y, \quad v = \Omega(x - b), \quad w = 0, \tag{2}$$

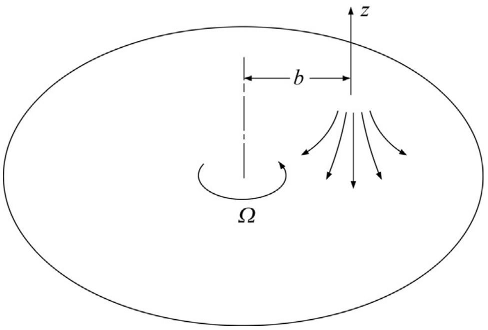

Figure 1 Configuration of the off-centered stagnation flow on a rotating disc [16,19].