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Analytical investigation of laminar flow through expanding or contracting gaps with porous walls

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Permeable channel

Abstract Laminar, isothermal, incompressible and viscous flow in a rectangular domain bounded by two moving porous walls, which enable the fluid to enter or exit during successive expansions or contractions is investigated analytically using optimal homotopy asymptotic method (OHAM). OHAM is a powerful method for solving nonlinear problems without depending to the small parameter. The concept of this method is briefly introduced, and it's application for this problem is studied. Then, the results are compared with numerical results and the validity of these methods is shown. After this verification, we analyze the effects of some physical applicable parameters to show the efficiency of OHAM for this type of problems. Graphical results are presented to investigate the influence of the non-dimensional wall dilation rate (α) and permeation Reynolds number (Re) on the velocity, normal pressure distribution and wall shear stress. The present problem for slowly expanding or contracting walls with weak permeability is a simple model for the transport of biological fluids through contracting or expanding vessels.

1. Introduction

In the heart of all different engineering sciences, everything shows itself in the mathematical relation that most of these problems and phenomena are modeled by ordinary or partial differential equations. Several authors used numerical methods like lattice Boltzmann method (LBM) [1–10], Control volume based finite element method (CVFEM)

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Nomenclature

$a^{\bullet}$	time-dependent rate
t	time
c	injection/suction coefficient
F	transformed of f
p	pressure
Δp_n	pressure drop in the normal direction
Re	permeation Reynolds number
u , v	velocity components along x, y axes, respectively
V_w	injection velocity

Greek symbols

υ	kinematic viscosity
α	non-dimensional wall dilation rate
τ	shear stress
ρ	fluid density

[11–24] and other methods [25–29] to solve the governing equations. In most cases, scientific problems are inherently of nonlinearity that do not admit exact solution, so these equations should be solved using special techniques. Some of them are solved using numerical techniques and some are solved using the analytical method of perturbation. In the numerical method, stability and convergence should be considered so as to avoid divergence or inappropriate results. In the analytical perturbation method, the small parameter is exerted to the equation. Since there are some limitations with the common perturbation method, and also because the basis of the common perturbation method is upon the existence of a small parameter, developing the method for different applications is very difficult. Therefore, some different methods have recently introduced some ways to eliminate the small parameter, such as the homotopy perturbation method [30–34], optimal homotopy perturbation method [35], variational iteration method [36,37], parameterized perturbation method [38,39], differential transformation method [40–43], homotopy analysis method [44,45], adomian decomposition method [46,47], least square method [48–51] and iteration procedure [52]. Optimal homotopy asymptotic method (OHAM) was considered for the solution of strongly nonlinear differential equations arising in heat transfer by Haq et al. [53]. They showed that this method provides a convenient way to optimally control convergence of the approximation series and adjust convergence regions. Hashmi et al. [54] used OHAM for finding the approximate solutions of a class of Volterra integral equations with weakly singular kernels. They showed that OHAM is a reliable and efficient technique for finding the solutions of weakly singular Volterra integral equations. Sheikholeslami et al. [55] analyzed using OHAM the problem of laminar viscous flow in a semi-porous channel in the presence of a transverse magnetic field. Their results provide further support that OHAM is a powerful tool for solving nonlinear differential equations. Sheikholeslami and Ganji [56] studied the magnetohydrodynamic flow in a permeable channel filled with nanofluid. They showed that that velocity boundary layer thickness decreases with increase of Reynolds number and nanoparticle volume fraction and it increases as Hartmann number increases. Marinca and Herisanu [57] used the optimal homotopy asymptotic method for solving Blasius equation. The applications of OHAM to other types of problems were given in [58–62].

Studies of fluid transport in biological organisms often concern the flow of a particular fluid inside an expanding or contracting vessel with permeable walls. Seepage across permeable walls is clearly important to the mass transfer between blood, air and tissue [63]. Therefore, a substantial amount of research work has been invested in the study of the flow in a rectangular domain bounded by two moving porous walls, which enable the fluid to enter or exit during successive expansions or contractions. Dauenhauer and Majdalani [64] studied the unsteady flow in semi-infinite expanding channels with wall injection. They are characterized by two non-dimensional parameters, the expansion ratio of the wall α and the cross-flow Reynolds number Re. Also flow and heat transfer in heat exchangers were studied by different authors [65–68]

In this study, optimal homotopy asymptotic method is applied to find the approximate solutions of nonlinear differential equations governing two-dimensional viscous flow through expanding or contracting gaps with permeable walls and have made a comparison with the Numerical Solution.

2. Flow analysis and mathematical formulation

Consider the laminar, isothermal and incompressible flow in a rectangular domain bounded by two permeable surfaces that enable the fluid to enter or exit during successive expansions or contractions. A schematic diagram of the problem is shown in Figure 1.

The walls expand or contract uniformly at a time-dependent rate $a^{\bullet}$. At the wall, it is assumed that the fluid inflow velocity V_w is independent of position. The equations of continuity and motion for the unsteady flow are

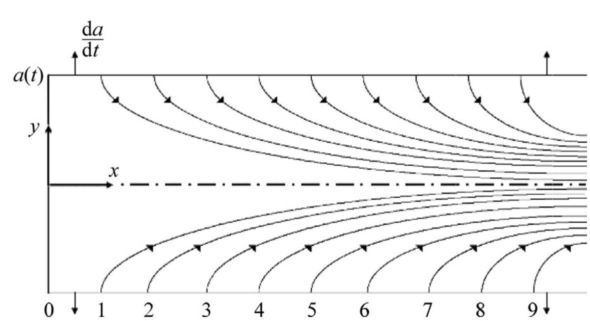

Figure 1 Two-dimensional domain with expanding or contracting porous walls.