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ORIGINAL ARTICLE

Numerical investigation of the flow of a micropolar fluid through a porous channel with expanding or contracting walls

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Abstract In this paper, we study the flow of a micropolar fluid in a porous channel with expanding or contracting walls. First, we use spectral collocation method on the governing equations to obtain an initial approximation for the solution of equations. Then using the obtained initial approximation, we apply the homotopy analysis method to obtain a recursive formula for the solution.

1. Introduction

Most nonlinear models of real-life problems are still very difficult to solve, either numerically or theoretically. In recent years, the investigation of the traveling wave solutions of nonlinear partial differential equations has begun to play an important role in the study of nonlinear physical phenomena. Nonlinear wave phenomena appear in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics and geochemistry. One of the most exciting advances of nonlinear science and theoretical physics has been the development of methods that look for exact solutions of nonlinear evolution equations. The availability of symbolic computational tools such as Mathematica and Maple has popularized the search for exact solutions of nonlinear equations. Several recent attempts have been made to develop new techniques for obtaining analytical solutions which reasonably approximate the exact solutions. Such techniques include the homotopy analysis method (HAM) [1–3], the Hirota method and Rimann theta function [4]. The HAM has been used successfully to solve a variety of nonlinear differential equations. However, the HAM suffers from

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a number of restrictive measures, such as the requirement that the solution sought ought to conform to the so-called rule of solution expression and the rule of coefficient ergodicity. In a recent study, Motsa and Sibanda et al. [5,6] proposed a spectral modification of the homotopy analysis method, the spectral-homotopy analysis method (SHAM) that seeks to remove some restrictive assumptions associated with the implementation of the standard homotopy analysis method.

The aim of this paper is to find solutions of the flow of a micropolar fluid through a porous channel with expanding or contracting walls. The study of the flow of a micropolar fluid in porous media has been active filed of research. Its theory (see Eringen [7,8]) derives from the need to model the flow of fluids that contain rotating micro-constituents. The rotation and gyration of the micro-constituents has a profound effect on the hydrodynamics of the flow [9] and the usual Navier-Stokes equations cannot adequately describe the motion of such fluids. The concept of micropolar fluid has been used in the investigation of various fluids, such as the flow of low concentration suspensions, liquid crystals [10], polymeric fluids and blood [11] as well as fluids with additives and turbulent shear flows [9].

2. Mathematical formulation

We consider the motion of an incompressible flow of a micropolar fluid between two contracting or expanding, porous disks and neglect the effects of body forces and body couples. Assuming the flow to be fully developed. The distance between the disks is $2a(t)$. The disks have the same permeability and expand or contract uniformly at a time-dependent rate $\dot{a}(t)$. The velocity component u, w are taken to be in the r, z direction, ϕ is the microrotation component, respectively. A geometry of the problem is given in Figure 1.

Under these assumptions, the continuity and the momentum equations are given by the following relations, respectively [12]:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

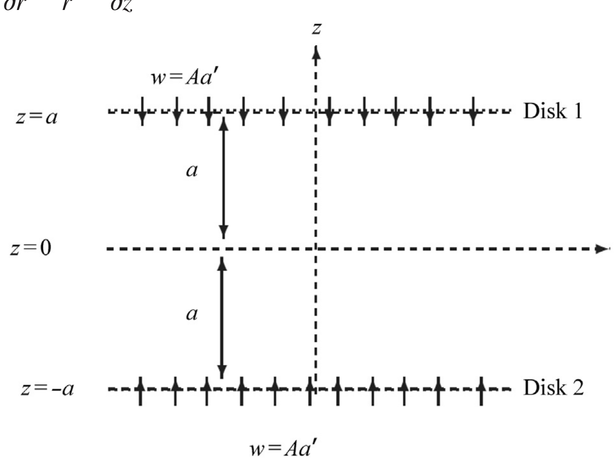

Figure 1 Geometry of the problem.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu+\kappa}{\rho}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{\kappa}{\rho}\frac{\partial \phi}{\partial z}, \tag{2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu+\kappa}{\rho}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) + \frac{\kappa}{\rho}\left(\frac{\partial \phi}{\partial r} + \frac{\phi}{r}\right), \tag{3}$$

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial r} + w\frac{\partial \phi}{\partial z} = \frac{\gamma}{j\rho}\left(\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} - \frac{\phi}{r^2}\right) + \frac{\kappa}{j\rho}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial r}\right) - 2\frac{\kappa}{j\rho}\phi, \tag{4}$$

Where ρ is the density, and j, γ, κ are the microinertial per unit mass, spin gradient viscosity and vortex viscosity, respectively. Here γ is assumed to be

$$\gamma = \left(\mu + \frac{\kappa}{2}\right)j,$$

in which μ is the dynamic viscosity and we take $j = a^2$ as the reference length. According to [13], we also assume that there is the strong concentration of microelements and the microelements close to the wall are unable to rotate. The problem has the following boundary conditions:

$$u = 0, \quad w = 2v_w = Aa, \quad \phi = 0; \quad z = a(t),$$
$$u = 0, \quad w = -2v_w = -Aa, \quad \phi = 0; \quad z = -a(t),$$

Where $A = 2v_w/\dot{a}$ is the measure of wall permeability [14]. We introduce the following similarity transformations that are motivated by the definition of the stream function (see Si et al. [15])

$$u = -\frac{1}{r}\frac{\partial \psi}{\partial z} = -\frac{\nu r}{2a^2}F_\eta(\eta, t), \quad w = \frac{1}{r}\frac{\partial \psi}{\partial r} = \frac{2\nu}{a}F(\eta, t), \tag{5}$$

$$\phi = -\frac{\nu r}{a^3}g(\eta, t), \quad \eta = \frac{z}{a}, \tag{6}$$

Where F is the dimensionless velocity and g is the dimensionless microrotation velocity. Substituting Eq. (5) and Eq. (6) into Eqs. (1)–(4) and eliminating pressure, one obtains the following nonlinear partial differential equations,

$$(1+K)F_{\eta\eta\eta\eta} + \alpha(3F_{\eta\eta} + \eta F_{\eta\eta\eta}) - 2FF_{\eta\eta\eta} - KG_{\eta\eta} - a^2\nu^{-1}F_{\eta\eta t} = 0, \tag{7}$$

$$\left(1+\frac{K}{2}\right)G_{\eta\eta} + \alpha(3G + \eta G_\eta) + KF_{\eta\eta} - 2KG + F_\eta G - 2FG_\eta - a^2\nu^{-1}G_t = 0, \tag{8}$$