Propulsion and Power Research 2013;2(3):201–207

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Propulsion and Power Research

www.sciencedirect.com

ORIGINAL ARTICLE

Chebyshev super spectral viscosity method for water hammer analysis

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Received 7 May 2013; accepted 17 June 2013
Available online 4 September 2013

KEYWORDS

Liquid propellant rocket engine (LPRE); Propellant transfer; Water hammer; Spectral method; Super spectral viscosity (SSV); Numerical simulation

Abstract In this paper, a new fast and efficient algorithm, Chebyshev super spectral viscosity (SSV) method, is introduced to solve the water hammer equations. Compared with standard spectral method, the method's advantage essentially consists in adding a super spectral viscosity to the equations for the high wave numbers of the numerical solution. It can stabilize the numerical oscillation (Gibbs phenomenon) and improve the computational efficiency while discontinuities appear in the solution. Results obtained from the Chebyshev super spectral viscosity method exhibit greater consistency with conventional water hammer calculations. It shows that this new numerical method offers an alternative way to investigate the behavior of the water hammer in propellant pipelines.

1. Introduction

Transients flow, also known as water hammer, is produced by a rapid change of flow velocity in the propellant pipelines that may be caused by sudden open or closure of valves, start or shutdown of pumps, rapid changes in demand condition, etc. This phenomenon may invalidate pipeline sealing and cause propellant leakage and even engine power loss. It is important to study both the water hammer phenomenon and the pressure transient discipline in order to reduce the effect of water hammer pressure and to design a reliable liquid rocket engine and more reasonable propellant system.

Transients flow in pipelines is formulated by a set of nonlinear, hyperbolic partial differential equations (PDE) formed by one-dimensional continuity and momentum equations. The equations can seldomly be solved analytically. Various numerical methods have been introduced for pipeline transient calculation. They include the method of characteristics (MOC) [1–3], finite difference (FD) [4,5], finite volume

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Peer review under responsibility of National Laboratory for Aeronautics and Astronautics, China.

ELSEVIER Production and hosting by Elsevier

2212-540X
http://dx.doi.org/10.1016/j.jppr.2013.07.004

Nomenclature

a	wave speed (unit: m/s)
c_i	coefficients to evaluate the first derivate matrix $\boldsymbol{D}^{(1)}$
d_{ik}	coefficients of matrix $\boldsymbol{D}^{(1)}$
$\boldsymbol{D}^{(1)}$	inner diameter of pipe (unit: m)
e	thickness of the pipe
E	Young's modulus of elasticity of the pipe (unit: Pa)
f	Darcy–Weisbach friction factor
g	acceleration of gravity (unit: m/s^2)
$h_i(x)$	Lagrange polynomials
K	bulk modulus of elasticity of the fluid (unit: Pa)
L	pipeline length (unit: m)
N	number of polynomials or collocation points in x
p	pressure (unit: Pa)
p_0	initial pressure in pipe (unit: Pa)
s	viscosity order
t	time (unit: s)
$u(x,t)$	unknown function
$\boldsymbol{u}$	vector of $u(x,t)$ evaluated at the collocation points
$\boldsymbol{u}^{(n)}$	vector of n order spatial derivative of $u(x,t)$ at the collocation points
$\hat{u}_k(t)$	time dependent expansion spectral coefficients
v	velocity (unit: m/s)
V	non-dimensional velocity
x	non-dimensional space coordinate
x_i	Chebyshev-Gauss-Lobatto points
$\bar{x}$	space coordinate (unit: m)
$T_k(x)$	Chebyshev polynomials of order k

Greek letters

δ_{ik}	Kronecker delta operator
ε	viscosity amplitude
$\phi_k(x)$	basis functions
η	coefficient
φ	non-dimensional pressure
ρ	density of fluid (unit: kg/m^3)
τ	non-dimensional time
η	coefficient

Subscripts

(n)	indicates n order derivative with respect to x
i,k	relative to the number of polynomials or collocation points

(FV) [6,7], and finite element (FE) [8,9]. Among these methods, MOC is proven to be the most popular one because of its numerical simplicity and computational efficiency. Unfortunately, the MOC approach becomes problematic when it is applied to complex systems or systems with varying parameters such as wave speed, material properties and multiples phase flow for the reason that the Courant number is not equal to unity. For these systems, MOC requires to use interpolation schemes or wave speeds/geometric adjustments. However, this would bring dispersion errors to the solutions.

To numerically solve water hammer partial differential equations, we need to compute the spatial derivatives first. Yet using the FD, FV or FE methods to compute them would typically require a large number of nodal points in order to yield satisfactory results. As the promising alternatives, the spectral and pseudo-spectral methods have been extensively used in recent years. The spectral methods differ from the FD, FV and FE methods in which global information is incorporated into the computing of a spatial derivative. The spectral methods can yield a smooth solution of greater accuracy with far fewer nodes and therefore less computational time than the FD, FV and FE methods. A wide variety of spectral schemes applied to fluid dynamics have been reviewed by Bashid et al. [10], Canuto et al. [11], Peyret [12], and Chen et al. [13].

It is well known that using spectral methods for nonlinear conservation laws would result in the occurrence of the Gibbs phenomenon once spontaneous shock discontinuities appear in the solution [14]. These spurious oscillations would in turn lead to loss of resolution and render the standard spectral approximations unstable. In this paper, we use the Chebyshev super spectral viscosity (SSV) method [15,16] to solve the water hammer equations. This method's specialty essentially consists in adding a spectral viscosity to the equations for the high wave numbers of numerical solution. This super spectral viscosity is sufficient to stabilize the numerical scheme when it is small enough to retain spectral accuracy. The numerical test, water hammer problem in a simple pipeline system, is considered. The numerical results were compared with the conventional water hammer calculations to demonstrate the accuracy of the method.

2. Mathematical formulation

2.1. Governing equations

For one-dimensional transient flow, based on the Newton's Second Law and the mass conservation principle, the basic control equations of transient flow which include the momentum conservation equation and continuity equation can be finally deduced and expressed as two partial differential equations [1].

$$\frac{\partial p}{\partial t}+v\frac{\partial p}{\partial \bar{x}}+\rho a^2\frac{\partial v}{\partial \bar{x}}=0,\quad \bar{x}\in[0,L] \tag{1}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial \bar{x}}+\frac{1}{\rho}\frac{\partial p}{\partial \bar{x}}+\frac{f}{2D}v|v|+g\cos\,\theta=0,\quad \bar{x}\in[0,L] \tag{2}$$

where, $\bar{x}$ is the space coordinate, t is the time, v is the velocity in pipe, p is the pressure, f is the Darcy–Weisbach friction factor, D is the inner diameter of pipe, ρ is the density of fluid, g is the acceleration of gravity, L is the pipeline length, and a is the wave speeds. The wave speeds