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Three-dimensional numerical simulations of vortex-induced vibrations of tapered circular cylinders

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ABSTRACT

Vortex-induced vibration (VIV) of a tapered cylinder is investigated numerically at a constant Reynolds number of 500 using three-dimensional numerical simulations. The objectives of the study is to identify the difference between the response of a tapered cylinder and that of a uniform cylinder. Simulations are conducted for the lengths and the taper ratios the same as those in the published experimental studies. Two cylinders are considered: one with a length to diameter ratio of 4.3 and a mass ratio of 2.27 and another one with a length to diameter ratio of 12.3 and a mass ratio of 6.1. Detailed analysis of the vibration amplitude and frequency, the vortex shedding flow mode and the lift coefficient are performed for the longer cylinder. It was found that the frequencies of the vortex shedding and the lift coefficient synchronize with the vibration frequency in the lock-in regime, but vary along the cylinder span outside the lock-in regime. For some reduced velocities, it is found that vortex shedding is in 2P mode at the small-diameter part and 2S mode at the larger-diameter part of the cylinder, forming a hybrid flow mode. The change of the flow mode on the cylinder span corresponds to the change of the phase difference between the lift coefficient and the displacement for about 180°. The lock-in regime of a tapered cylinder is found to be wider than that of an equivalent uniform cylinder.

1. Introduction

Vortex-induced vibration (VIV) of bluff body structures are one of the classical design issues in fluid engineering as strong vibrations may lead to catastrophic structural damages. In the offshore engineering, the sea currents can similarly induce large-amplitude vibrations of offshore structures, such as platforms and underwater pipelines. Due to this reason, VIV of cylindrical structures has attracted much attention of engineers and scientists and been studied extensively in the past decades. Most of the studies are focused on the VIV of an elastically mounted rigid cylinder in a fluid flow. The detailed review of the research on VIV of an elastically mounted cylinder can be found in Sarpkaya [15], Sumer and Fredsoe [19], Bearman [2], Williamson and Govardhan [21,23].

The response of an elastically mounted rigid cylinder in a fluid flow has been the topic of many studies because this simple case provides fundamental mechanisms of VIV. When VIV of a cylinder is studied, the velocity of the fluid flow is commonly nondimensionalized as $V_r = U/f_nD$, where U, f_n and D are the free-stream velocity, the natural frequency and the diameter of the cylinder, respectively. This nondimensional velocity is called reduced velocity. It has been found that the vortex shedding frequency and the vibration frequency synchronize in a range of reduced velocity. The synchronization between the vortex shedding and the vibration is also called lock-in [7]. In the lock-in range of the reduced velocity, the vortex shedding frequency does not follow the Strouhal law. The lock-in range of the reduced velocity is also found to be dependent on the mass ratio m^* defined as $m^* = m/m_d$, where m and m_d are the mass of the cylinder and the displaced fluid mass by the cylinder, respectively. By analyzing the experimental data, Khalak and Williamson [7] found that the maximum response amplitude is dependent on the product of the mass ratio m^* and the damping ratio ζ, and the lock-in range of the reduced velocity is primarily dependent on the mass ratio.

Due to its efficiency, two-dimensional (2D) simulations of VIV of a cylinder at very low Reynolds numbers in the laminar flow regime have been popularly used to investigate VIV [12,18,10,3,29,30]. The maximum amplitudes of the response in these low-Reynolds-number simulations are lower than those measured in the high-Reynolds-number laboratory studies. Some three-dimensional simulations of VIV in the turbulent flow regimes

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have also conducted, but still at relatively low Reynolds numbers in the turbulent wake flow regime [11,13,31]. The 3D numerical simulations for Re = 1000 by Lucor et al. [11] and Zhao et al. [31] predicted the lock-in range of reduced velocity well but underestimate the maximum vibration amplitude compared with the experimental data, which were measured at much higher Reynolds numbers.

The studies of VIV of a circular cylinder in a fluid flow have been extended to various cases that are relevant to the engineering. For example, VIV of a cylinder close to a plane boundary has been studied due to its relevance to the subsea pipelines close to the sea floor [5,24,25,28] and VIV of a truncated cylinder, which is relevant to the floating offshore platform, was studied numerically by Zhao and Cheng [32].

The interaction of a spanwise shear flow with a uniform cylinder or the interaction of uniform flow with a tapered cylinder have also attracted much attention recently. The flow patterns of the two cases (a uniform cylinder in a shear flow and a tapered cylinder in a uniform flow) were found to be similar to each other. When a uniform circular cylinder is placed in a spanwise shear flow, spanwise cellular wake is the typical wake flow structure and vortex shedding frequency changes at the boundary between two cells [1,8,17,14]. The cellular wake flow was also found in the wake of a tapered cylinder in a uniform flow [1]. The VIV of a uniform cylinder in a spanwised shear flow has been studied numerically by Zhao [33] and it was found that in the lock-in regime, the vortex shedding frequency and the frequency of the lift coefficient synchronize along the cylinder span. By conducting experimental studies of forced vibration of a tapered cylinder in a uniform flow, Techet et al. [20] also found the synchronization of the vortex shedding frequency along the cylinder span in a certain range of the vibration frequency of the cylinder. Zeinoddini et al. [26] investigated VIV of a tapered cylinder in a uniform flow experimentally and found that the lock-in range of a tapered cylinder is wider than that in its equivalent uniform cylinder. Seyed-Aghazadeh et al. [16] conducted an experimental study of VIV of tapered cylinders with a constant mass ratio and a constant cylinder length and found that the maximum amplitude of the vibration stayed almost constant for all the taper ratios.

Although some experimental studies of VIV of a tapered cylinder have been conducted, they are mainly focused on the vibration amplitude and frequency of the cylinder and their dependence on the taper ratio. Little attention has been paid on the vortex shedding flow patterns of a free-vibrating tapered cylinder, which is important to understand the mechanisms of VIV. In this study, VIV of an elastically mounted tapered cylinder in a uniform flow is investigated numerically. The main objectives of this study are to identify the difference between the lock-in regimes of a tapered and a uniform cylinder and to study the wake flow patterns for VIV of a tapered cylinder in the lock-in regime. The parameters used in this study are chosen to be as close to those used by Zeinoddini et al. [26] as possible. In the numerical model, the three-dimensional incompressible Navier-Stokes (NS) equations are solved to predict the fluid flow and the equation of motion is solved to predict the response displacement of the cylinder.

2. Numerical method

The vortex-induced vibrations of a tapered cylinder is considered in this study as shown in Fig. 1. The rigid cylinder is elastically mounted in a uniform flow and allowed to vibrate only in the cross flow direction. The diameter of the midsection of the tapered cylinder is defined as D. The taper ratio of the cylinder is defined as $\alpha = (D_2 - D_1)/(2L)$where D_2 and D_1 are the maximum and minimum diameters at the two ends of the cylinder, respectively. The reduced velocity is defined as $V_r = U/f_n D$, where U is the free

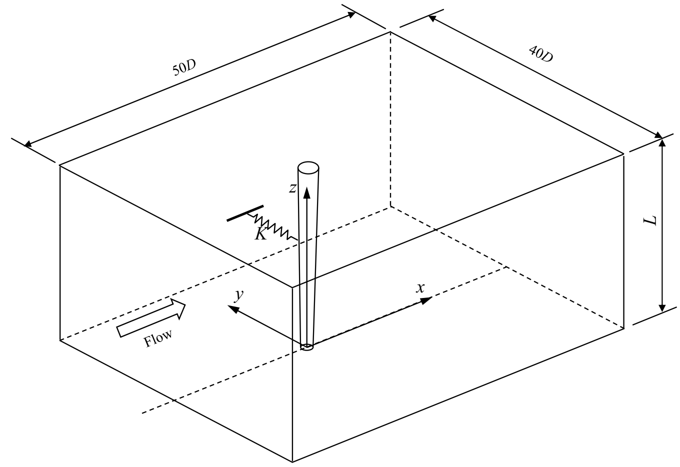

Fig. 1. Sketch for VIV of a tapered cylinder in a uniform flow.

stream velocity and (f_n) is the structural natural frequency measured in the vacuum. The Reynolds number is defined as Re = UD/ν, where ν is the kinematic viscosity of the fluid. The flow around the tapered cylinder is simulated by Direct Numerical Simulation (DNS), where the unsteady three-dimensional incompressible Navier-Stokes (NS) equations are solved. Instead of using turbulent models to approximately predict the turbulence, the whole range of spatial and temporal scales of the turbulence must be resolved in DNS. Due to the limitation of the computer power, DNS has been generally used to simulate vortex shedding flow and VIV of a cylinder in the low Reynolds numbers, where the turbulent scale is relatively large. The Arbitrary Lagrangian Eulerian (ALE) scheme is applied to simulate the flow around a vibrating cylinder. The ALE scheme allows the computational mesh to move according to the moving boundary, but not necessary with the fluid particles. In the ALE scheme, the incompressible NS equations are written as

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + \left(u_j - \hat{u}_j\right) \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} \tag{2}$$

where x_i (x_1–x_3 = x–z, respectively) is the Cartesian coordinate, u_i is the velocity in the x_i-direction, ρ is the fluid density and $\hat{u}_j$is the velocity of the moving mesh in the x_j-direction. The motion of the cylinder is predicted by the equation motion

$$m\ddot{Y} + c\dot{Y} + KY = F_L \tag{3}$$

where Y, $\dot{Y}$ and $\ddot{Y}$ are the displacement, velocity and the acceleration of the cylinder, respectively, c and K are the damping coefficient and the stiffness of the spring, respectively, and F_L is the lift force of the cylinder.

The NS equations are discretized using the Petrov-Galerkin Finite Element Method (PG-FEM), which was proposed by Brooks and Hughes [4]. By using the fractional step method in the temporal discretization, Zhao et al. [27] developed the three-dimensional PG-FEM formulae for solving the NS equations, which has been used in a number of studies of VIV of circular cylinders [31–33]. The detail of the PG-FEM formulae of the NS equation can be found in Zhao et al. [27] and will not be repeated here. The equation of motion (3) is solved by the fourth-order Runge-Kutta method.

A 50D long, 40D wide computational domain is used in the numerical simulation as shown in Fig. 1, with the cylinder being located 20D downstream the inlet boundary. The height of the computational domain is the same as the cylinder length. The boundary condition for the velocity at the inlet boundary for a uniform flow