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# Non-linear evolution of uni-directional focussed wave-groups on a deep water: A comparison of models

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ABSTRACT

Up until the point at which ocean waves break, their dynamics are generally assumed to be accurately modelled by potential flow theory. For practical and computational reasons it is often useful to approximate the full potential flow solution with bandwidth and amplitude limited equations. A approximation used for waves on deep water is the Broad-banded Modified Non-linear Schrödinger equation (also known as the modified Dysthe equation). In this paper we compare this approximate model with potential flow simulations of focussing uni-directional wave-groups. We find that for moderate non-linearity the approximate model predicts very similar changes to the potential flow model. However, one of the dominant non-linear changes to the wave-group is a localised increase in the bandwidth and contraction in physical length, and beyond a certain point the approximate model fails to accurately reproduce this causing other elements, such as the maximum wave amplitude, to be poorly modelled. This modelling inaccuracy occurs in cases where, based on the initial conditions of the simulation, the approximate model would be expected to be accurate.



## 1. Introduction

The evolution of ocean waves is a weakly non-linear phenomenon until close to the point at which waves break. A variety of models have been proposed to describe this evolution. In this paper we assume that the evolution of waves (up until breaking) can be accurately described by numerical solutions to the potential flow equations using standard boundary conditions at the free surface which we describe as the 'fully non-linear' model. We compare fully non-linear results to numerical solutions an approximate model which simulates the evolution of the complex wave envelope using higher order extensions of the non-linear Schrödinger (NLS) equation.

Clamond et al. [1] carried out a comparison of the different models examined in this paper but their work focussed on long time-scales rather than the detailed structure of waves locally tall and spatially concentrated wave groups as studied here. Comparisons were also made between the NLS and potential flow simulations by Henderson et al. [2]. Comparisons of the higher order non-linear Schrödinger equation to experiments were made by Shemer and Dorfman [3] and Lo and Mei [4]. A similar study to ours – comparing the approximate envelope model against an exact numerical model – was carried out by Shemer et al. [5] comparing the Zakharov equations, broadbanded NLS, and experiments. This later paper found good agreement for narrow bandwidths between the Zakharov equations and the broadbanded NLS and explored the bandwidth limitations of this. Our study pursues this theme, comparing results from the broadbanded NLS equation against those from a high order spectral scheme for the potential flow equations.

To compare the two models we run simulations of focussed wave-groups. Studying the non-linear changes to isolated wave-groups has been used to investigate non-linear wave evolution by numerous authors using physical experiments [6,7], numerical models [8–10], and analytically [11,12]. In uni-directional waves non-linearity leads to significant changes to the shape of the wave-group – relative to linear evolution non-linear groups become taller and narrower with the largest wave in the group moving towards the front of the wave-group. An example of this is shown in Fig. 1. The formation of this very sharp peak in both the wave envelope and also an individual crest may be compared to the prediction by Lighthill [13] for the changes in both local amplitude and wavenumber as a modulated Stokes wave train evolves. Lighthill predicted the formation of a cusp in the wave envelope; his analysis being based on Whitham's theory [14] for nonlinear-systems where frequency dispersion and amplitude dispersion would be expected to be in competition.

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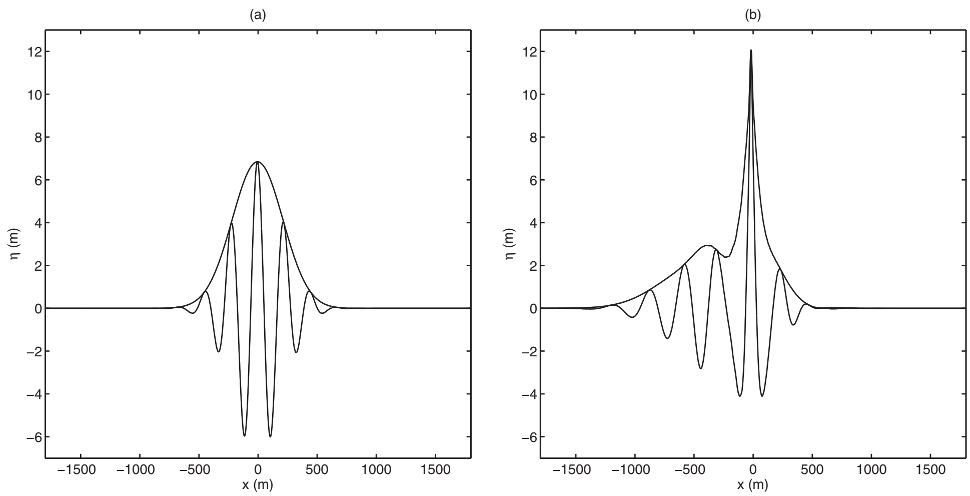


**Fig. 1.** Example of the non-linear changes to a wave-group using fully non-linear model. Both free surface and envelope shown with bound harmonics removed. (a) Linear and (b) fully non-linear.

This paper only examines the evolution of uni-directional waves. In the real ocean deep-water waves are not uni-directional but are directionally spread. This fundamentally changes the non-linear processes by which large waves form [15] and means that great caution must be used when applying the results of uni-directional modelling to real ocean waves. In particular, the extra amplitude observed in uni-directional waves is greatly reduced in waves with a realistic directional spreading. Nevertheless, uni-directional waves are an important limiting case for the evolution of real ocean waves and are often used in numerical and physical experiments for practical reasons. An extension of the results here to directionally spread wave-groups is presented in Adcock and Taylor [16].

## 2. Method

In this paper we compare fully non-linear potential flow simulations with results solving the broadbanded modified non-linear Schrödinger equation. Our test cases study the focussing of isolated wave-groups on deep water.

We take as our initial conditions a wave-group which, under linear evolution would form a 'NewWave' wave-group 80 periods later. The NewWave is the expected shape of a large wave in a random sea state (see Lindgren [17] and Boccotti [18]) and, at 'focus' ($t = 0$) is given by

$$\eta(x) = a\frac{\sum_n S(k_n)\cos(k_n x)}{\sum_n S(k_n)}, \tag{1}$$

where $S(k_n)$ is the discretised wavenumber spectrum of the underlying sea-state and $a$ is the amplitude of the wave-group which can specified arbitrarily. The shape of the linear NewWave group is shown in Fig. 1a. This approach has been widely used to study non-linearity of large waves and Adcock et al. [19] found that analysis of isolated wave-groups closely matched the non-linear changes observed to large waves in random wave-fields.

For these simulations we use a Gaussian wavenumber spectrum which has the shape

$$S(k) = \lambda \exp\left(\frac{-(k-k_p)^2}{2s_x^2}\right), \tag{2}$$

where $k_p$ is the peak wavenumber and $s_x$ is the bandwidth. The parameter $\lambda$ scales the spectrum although as we only use the shape of the spectrum in this paper this factor does not reappear. In this study we use $k_p = 0.0279\,\mathrm{m}^{-1}$ (a wavelength of 225 m) and $s_x = 0.0046\,\mathrm{m}^{-1}$. The spectral bandwidth is chosen by fitting the spectral peak of a JONSWAP spectrum with $\gamma = 3.3$ with the lower amplitude high frequency tail removed. As such, this bandwidth is of the same order of magnitude as would be found in a winter storm in the North Sea. To classify the runs we use the non-dimensionalised amplitude ($ak_p$) that the wave-group would reach under linear evolution. The most non-linear case we consider would have focussed with $ak_p = 0.18$. Allowing for the truncation of the high spectral tail, a wave of this steepness might be appropriate for a 1 in 1000 wave in a limiting steepness sea-state. Fig. 2 in Socquet-Juglard et al. [20] demonstrates that a limiting steepness of $H_S k_p = 0.36$ occurs in the northern North Sea. Assuming a Rayleigh-type linear crest distribution in a severe sea-state and the truncation of the upper spectral tail, a maximum NewWave individual steepness of $ak_p = 0.18$ is plausible.

The numerical solution to the potential flow equation uses the high-order pseudo-spectral numerical scheme developed by Bateman et al. [21]. The results used in this paper are taken from Gibbs [22] where they are analysed in depth. As with any numerical solution the results will not be exact, however great care was taken with the simulations and for the purposes of this paper we assume that the results of these simulations can be taken as a benchmark to compare against approximate models. The results in this paper used a spatial resolution of seven times the peak wavenumber, a time step of 0.02 s with an Adams–Bashford scheme with the equivalent peak wave period $T_p = 12$ s, this gives 97 time-steps per period. The length of the ocean in the assumed periodic computational domain is 8.0 km with 1024 spatial points. A 7th order expansion of the Dirichlet–Neumann G-operator is used [21]. Some filtering was applied to the highest wavenumber using the 5-point smoothing function of Dommermuth and Yue [23].

Higher order extensions to the non-linear Schrödinger equation were derived in a series of papers by Dysthe and Trulsen [24,25]. The equations model the complex wave envelope, $A$, of the freely propagating waves. The free surface may be recreated from the linear components and narrow bandwidth approximations to the 2nd and 3rd order bound components

$$\eta = \Re(\eta_{linear} + \eta_{2-} + \eta_{2+} + \eta_3), \tag{3}$$

where

$$\eta_{2-} = \frac{1}{2\omega}\frac{\partial \phi}{\partial x} - \frac{1}{16k}\frac{\partial^2 |A|^2}{\partial x^2}, \tag{4}$$

$$\eta_{linear} = A\exp(i(kx - \omega t)), \tag{5}$$