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Numerical investigation of oscillations within a harbor of parabolic bottom induced by water surface disturbances

Dong Shao [a,*], Xi Feng [b], Weibing Feng [a]

[a] College of Harbor Coastal and Offshore Engineering, Hohai University, Nanjing 210098, China
[b] Civil and Coastal Engineering Department, University of Florida, Florida, FL 32611, USA

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ABSTRACT

Oscillations within a rectangular harbor of parabolic bottom induced by water surface disturbances are investigated numerically based on Boussinesq equations and results are used to reveal the characteristics of the oscillations generated by disturbances of this type. The similarities and differences compared with those generated by a movable seafloor are also discussed. Relatively local and small-scale water surface disturbances may induce obvious transverse oscillations with little trace of longitudinal ones. The predominant transverse components are those with small alongshore mode number m and no node in the offshore direction. The augmentation of the rapidity of depth variation of the parabolic bottom may shift the resonant frequencies to larger values. These transverse modes are sensitive to the initial position of water surface disturbances. The spatial structure of each mode is well captured by the existing analytical solution based on shallow water equations. Although longitudinal oscillations may not be steadily generated with water surface disturbances, some patterns of several low-mode ones occur and are also sensitive to the position of the disturbances. Wavelet spectra are used to analyze their evolutions and comparisons are made with theoretical predictions for the three principle longitudinal modes.

1. Introduction

Harbor oscillations may cause many problems such as preventing of cargo operations, breaking of mooring ropes, damaging of infrastructures or moored vessels and can be triggered by the match of the eigenvalues of the free oscillations of a harbor and the external forces coming from wave groups, atmospheric pressure disturbances, landslides or shear flows etc.

Studies of harbor oscillations are firstly carried out analytically on a constant depth on some harbor geometries like a rectangular harbor, a basin with an entry channel [1,2], multiple basins or a coupled bay-river system [3,4]. Based on the understanding of the physical mechanism of harbor oscillations achieved by these early work, formulations are developed to describe the response of harbors with different depths [5,6]. For a rectangular harbor with a constant slope, Wang et al. [7] presented an explicit formula of longitudinal oscillations based on the linear shallow water approximation and revealed that the oscillation amplitudes and the positions of nodes within the harbor are influenced by the slope. The transverse oscillation modes due to refraction of the waves on the parabolic bottom and the dispersion relationship are also discussed. Following them, Shao et al. [8] derived the formulations for a harbor of parabolic bottom. With the analytic solutions, Wang et al. [9] investigated numerically the oscillations induced by seafloor movements on a constant slope based on a Boussinesq type model and observed that small-scale seafloor movements usually generate very weak longitudinal oscillations but evident larger transverse ones which are sensitive to the location of the movable seafloor. Relatively larger-scale seafloor movements containing more energy are found possible to induce larger longitudinal oscillations. However, in addition to seafloor movements, water surface disturbances within the harbor, although embody less energy, are much more frequently encountered and can be caused easily by floating body movements, changing atmospheric pressures, dock failures etc. at different locations [10,11]. As another type of disturbance acting on the water body inside the harbor, water surface disturbances may manifest different behaviors and have different properties.

The analytical descriptions derived by Shao et al. [8] are summarized in Section 2 with a brief introduction of the Boussinesq type model MIKE 21 BW [12] used for the simulations. In Section 3, numerical investigations are presented on the oscillations induced by water surface disturbances. Effects of variations of the bottom parameter (the rapidity of depth variation) of the parabolic bottom

* Corresponding author.
E-mail address: shao20130905@163.com (D. Shao).

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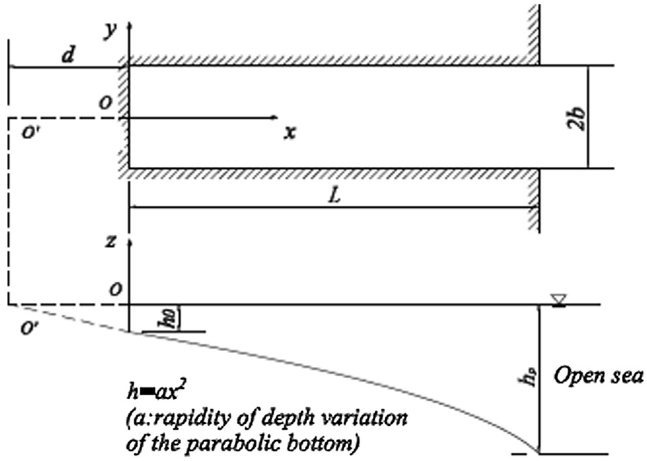

Fig. 1. Definition sketch of the harbor and coordinate system of the analytic solutions of Shao et al.

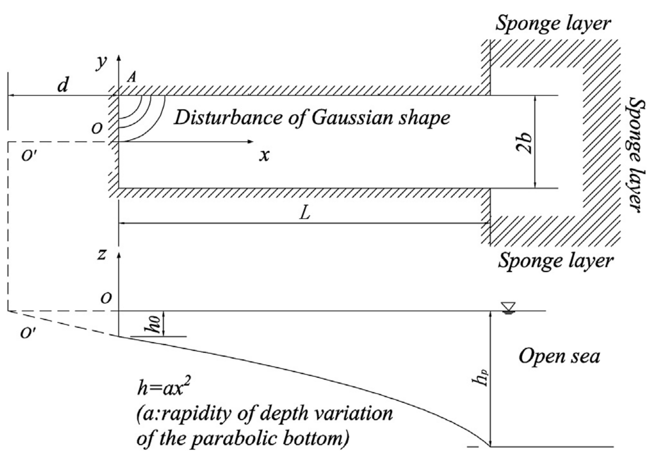

Fig. 2. Definition sketch of the rectangular harbor of parabolic bottom with water surface disturbance of Gaussian shape at the corner.

and the location of the disturbances are examined. Conclusions are drawn in the last section.

2. Analytic solutions of oscillations within a rectangular harbor of parabolic bottom and the numerical model applied

2.1. Analytic solutions of oscillations within a rectangular harbor of parabolic bottom

A brief review of analytic solutions derived by Shao et al. [8] for both longitudinal and transverse oscillations in a rectangular harbor of parabolic bottom is given in this section and the details of the formulations can be found in their work. For a rectangular harbor with the long axis in the positive x direction shown in Fig. 1, the backwall and the entrance are located respectively at $x = d$ and $x = d + L$. Assuming a horizontal seafloor in the open ocean, the water depth inside and outside of the harbor is given by:

$$h(x, y) = \begin{cases} h_0 + ax^2 d \le x \le L; \\ h_p x > L \end{cases} \tag{2.1}$$

where a describes the rapidity of the variation of the bottom and h_0 is the water depth at the origin of the x axis. When $h_0 > 0$, it denotes mathematically a so-called non-idealized parabolic bottom while $h_0 = 0$ means a so-called idealized parabolic bottom. For a non-idealized parabolic bottom with $h_0 > 0$, the backwall can be situated at $x = 0$ where the water depth is h_0 and for an idealized bottom, $x = 0$ is where the extended virtual bottom and mean sea level intersect and the backwall may be placed at a certain distance from the origin of the x axis to assure some positive water depth there. The width of the harbor is $2b$ from $y = -b$ to $y = b$ while the shoreline runs in the y direction (the cross harbor direction). The z coordinate is positive upward from the mean water level. The open sea outside the harbor has a constant depth that equals h_p.

The solution of longitudinal oscillations based on linear shallow water approximation for a non-idealized parabolic bottom can be described as

$$\zeta^L = C_1 P_\nu(\tau) + C_2 Q_\nu(\tau) \tag{2.2}$$

where P_ν and Q_ν are zero order Legendre functions of the first and second kinds respectively with $\tau = \frac{x}{i\sqrt{h_0/a}}$. The two constants C_1 and C_2 are given as

$$C_1 = \frac{2A_0 Q_\nu'(\tau_0)}{Q_\nu'(\tau_0)\left(P_\nu(\tau_1) + \mu_H P_\nu'(\tau_1)\right) - P_\nu'(\tau_0)\left(Q_\nu(\tau_1) + \mu_H Q_\nu'(\tau_1)\right)} \tag{2.3}$$

$$C_2 = \frac{2A_0 P_\nu'(\tau_0)}{P_\nu'(\tau_0)\left(Q_\nu(\tau_1) + \mu_H Q_\nu'(\tau_1)\right) - Q_\nu'(\tau_0)\left(P_\nu(\tau_1) + \mu_H P_\nu'(\tau_1)\right)} \tag{2.4}$$

where $\tau_0 = \frac{d}{i\sqrt{h_0/a}}$, $\tau_1 = \frac{L}{i\sqrt{h_0/a}}$, $\mu_H = \frac{\omega b}{g\sqrt{h_0/a}}\left(1 + \frac{2i}{\pi} ln \frac{\gamma kb}{\sqrt{2e^{3/2}}}\right)$ and $\nu = \frac{1}{2}\left(\sqrt{1 - \frac{4\omega^2}{ga}} - 1\right)$. $\gamma = 1.7810724$ is the exponential of Euler's constant. A_0 is the amplitude of the incident wave, k is the wave number, ω is the angular frequency of the longitudinal oscillation and g is the acceleration of gravity. The apostrophe represents first derivative with respect to τ. For an idealized bottom with $h_0 = 0$, the solution of longitudinal oscillations can be simplified as

$$\zeta^L = \begin{cases} D_1\sqrt{x} + D_2\sqrt{x} ln x & \delta = 0; \\ D_1 x^{\frac{-1+\sqrt{\delta}}{2}} + D_2 x^{\frac{-1-\sqrt{\delta}}{2}} & \delta > 0; \\ D_1 \dfrac{\cos\left(\frac{\sqrt{-\delta}}{2} ln x\right)}{\sqrt{x}} + D_2 \dfrac{\sin\left(\frac{\sqrt{-\delta}}{2} ln x\right)}{\sqrt{x}} & \delta < 0 \end{cases} \tag{2.5}$$

where $\delta = 1 - \frac{4\omega^2}{ga}$ and $x \in [d, L]$. The constants D_1 and D_2 may be determined with the boundary conditions at the backwall and at the entrance of the harbor.

The transverse oscillations ζ^T are excited when the transverse wavelength shares a specific relationship with the harbor width $2b$ in terms of wave number $k_{m,n}$ which is described by

$$k_{m,n} = \frac{m\pi}{2b}, m = 1, 2, 3, \ldots \tag{2.6}$$

while there are m possible transverse modes, the modes with small m may be most important because of their relatively long wavelength.

Transverse oscillations within the rectangular harbor of idealized parabolic bottom can be given as

$$\zeta^T = C_3\left(\frac{\tau}{k_{m,n}}\right)^{-1/2} K_\nu(\tau) \tag{2.7}$$