Applied Ocean Research 59 (2016) 165–182

Contents lists available at ScienceDirect

Applied Ocean Research

journal homepage: www.elsevier.com/locate/apor

Nonlinear hydrostatic analysis of flexible floating structures

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A R T I C L E I N F O

Article history:
Received 23 July 2015
Received in revised form 25 May 2016
Accepted 31 May 2016
Available online 23 June 2016

Keywords:
Hydrostatic analysis
Floating structures
Updated Lagrangian formulation
Finite element method
Hydrostatic experiment

A B S T R A C T

In this study, we present an updated Lagrangian Finite Element (FE) formulation for a geometrically nonlinear hydrostatic analysis of flexible floating structures subjected to buoyancy, self-weight, and various external static loads. The nonlinear equation is linearized with respect to a reference configuration and the resulting FE formulation is iteratively solved using the Newton-Raphson method. The initial stress effect, normal vector change, and buoyancy change are comprehensively considered in the tangential stiffness term of the hydrostatic equations. A special numerical integration technique is developed to handle the wet-surface change without re-meshing. Through the proposed numerical method, the hydrostatic equilibrium can be easily calculated considering various static and quasi-static loading conditions and the stress field of elastic bodies can be more accurately evaluated in the case of large displacement. Various nonlinear hydrostatic problems are solved to demonstrate the general capability of the proposed method. In particular, a hydrostatic experimental test was performed and the results are compared with those obtained using the proposed numerical method.

1. Introduction

In ocean environments, floating structures such as ships, offshore platforms, and offshore facilities are always subjected to various hydrostatic and quasi-static loads (e.g. structural weight, ballast water weight, and cargo weight) [1]. Calculating the hydrostatic equilibrium [2–9] is basic and important for analyzing the stability and the strength of floating structures. Also, in 3D hydroelastic analyses [10–16], a hydrostatic analysis has become a prerequisite to obtain the hydrostatic equilibrium and stress fields required for constructing the complete hydrostatic stiffness [10–12,16] whereas it is not essential for 2D hydroelastic analyses [17–23].

For a long time, the hydrostatic equilibrium has been calculated based on the rigid body assumption using pressure integration techniques [2–9], where floating structures are assumed to be rigid. While those methods are simple, they are not always applicable to flexible structures and require additional numerical operations (such as pressure projection) to calculate the stress fields of the floating structures caused by the hydrostatic pressure.

Nevertheless, it is hard to find methods to accurately calculate the hydrostatic equilibrium (and stress fields) of flexible floating structures. Basically, the hydrostatic analysis of floating structures could be nonlinear, mainly because of large displacement and wet-surface change. Furthermore, when floating structures are modeled using finite elements, difficulty in numerical integration of partially wetted elements arises from non-matching between the finite element mesh and the free surface. Such non-matching mesh problems frequently occur in the analysis of fluid-structure interaction problems (i.e. transient sloshing in elastic tanks) and proper treatment [24–26] is an important issue in numerical analyses.

The objective of this study is to develop a general method to calculate the hydrostatic equilibrium of 3D flexible floating structures, with the aim of accurately evaluating the draft and the stress fields of these structures. We employ the updated Lagrangian finite element formulation [27,28] for the nonlinear hydrostatic analysis of flexible floating structures. After linearizing nonlinear terms, we obtain incremental equilibrium equations, which are iteratively solved using the Newton-Raphson method. Wet-surface change, normal vector change, and buoyancy change due to structural displacement are completely considered. To efficiently handle the non-matching mesh

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http://dx.doi.org/10.1016/j.apor.2016.05.016
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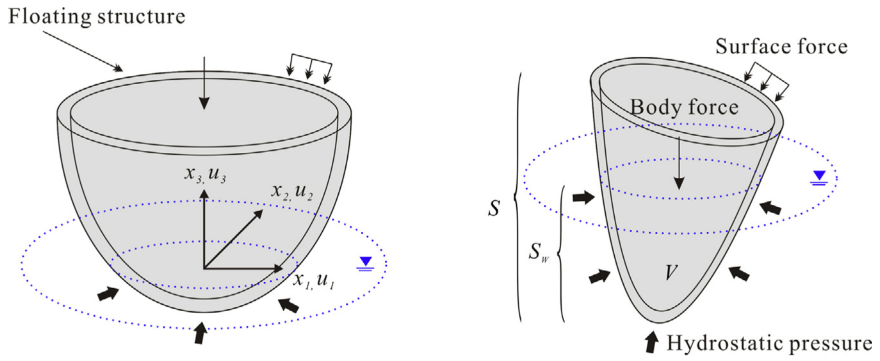
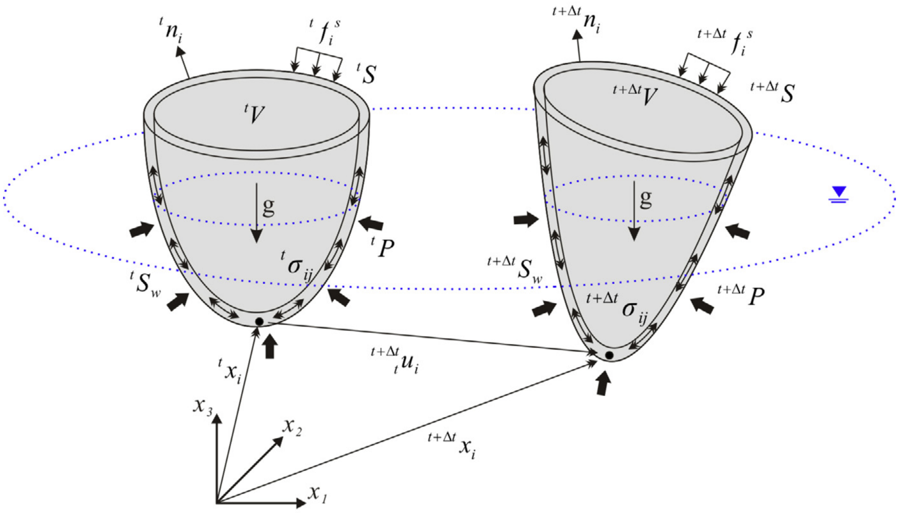


(a) Initial state (b) Hydrostatic equilibrium

Fig. 1. Hydrostatic analysis of a flexible floating structure.



(a) Configuration at time t (b) Configuration at time t+Δt

Fig. 2. Two configurations of a flexible floating structure at different time instances.

problem without re-meshing, a special numerical integration technique is developed. The proposed formulation and numerical method can be used for the hydrostatic analysis of both rigid and flexible floating structures.

In Section 2, the incremental equilibrium equations are presented. The finite element discretization procedure and the equations for a rigid body analysis are derived in Section 3. In Section 4, an effective numerical integration technique is developed. In Section 5, the feasibility of the proposed numerical procedure is demonstrated through various nonlinear hydrostatic problems corresponding to rigid and elastic body cases. In Section 6, a hydrostatic experimental test that has been performed is described and the results are compared with those obtained using the proposed numerical method.

2. Incremental equilibrium equations

As shown in Fig. 1(a), an undeformed three-dimensional (3D) flexible structure is initially floating in calm water and a fixed Cartesian coordinate system (x_1, x_2, x_3) is introduced. Note that, at the initial state, the floating structure can be arbitrarily positioned because the configuration in the hydrostatic equilibrium is unknown. Through a nonlinear hydrostatic analysis, we can obtain the hydrostatic equilibrium, where the external forces (e.g. surface force, body force, and hydrostatic pressure) are balanced, as shown in Fig. 1(b). The volume and the surface of the floating structure are denoted by V and S, respectively. The hydrostatic pressure is applied on the wet-surface, S_W. The structural material is assumed to be homogeneous, isotropic, and linear elastic.

Since the difference between the configurations at the initial state and in the hydrostatic equilibrium is not small in general, large displacement should be properly considered in the analysis procedure. Of course, when floating structures are damaged and grounded, large displacement can also occur.

The incremental equations for the freely floating flexible structure are obtained through the updated Lagranagian formulation [27,28]. In the following formulations, superscripts (or subscripts) t and $t + \Delta t$ are adopted to denote time. In a nonlinear hydrostatic analysis, t and $t + \Delta t$ are dummy variables to indicate incremental variables rather than the actual time as in dynamic analyses [27].

In Fig. 2, two configurations of the flexible floating structure at different time instances are shown, and they are denoted by the left superscripts t and $t + \Delta t$, respectively. The components of the material point vectors for the floating structure in the configuration at time