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Boundary conditions for modeling scattered wave field around floating bodies in elliptic wave models

Claudia Cecioni*, Giorgio Bellotti

Engineering Department, University of Roma TRE, via Vito Volterra 62, 00146 Rome, Italy

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ABSTRACT

This paper presents a new methodology to reproduce the interaction between waves and floating objects over mildly variable bathymetry. The elliptic mild slope equation solves the fluid velocity potential over a domain, which does not include the near field around each object. The waves scattered by the floating bodies are considered by means of boundary conditions at the edges of the near field areas. The coefficients of these boundary conditions are obtained nesting the solution of a near field three dimensional solver with the elliptic model solution. Comparison with a 3D numerical model, which solves the potential flow field over the whole domain, is used to validate the proposed approach. An example of application to reproduce the wave field around an array of wave energy converters is presented.

1. Introduction

The wave field in an open sea with the presence of floating objects is composed by incident, diffracted and radiated waves. The scattering of an incident wave field by a group of bodies, may lead to wave forces on each of them that significantly differ from those applied on an isolated body. The detailed reproduction of multiple floating objects requires accurate modeling of the hydrodynamic interaction problem, and, given the complex flow field, usually requires three-dimensional (3D) analysis. When dealing with arrays of wave energy converters (WEC hereinafter), the need to perform several simulations in order to optimize the WECs layout in terms of wave energy production, cannot be guaranteed at reasonable computational costs with 3D models covering the whole area of interest.

Beels et al. [1] developed a numerical model, which solves the time-dependent mild-slope equation, to reproduce the wave field around single or multiple WECs. The issue of reducing the computational effort on simulating large sea areas in the presence of wave energy converter devices has been obtained also by coupling wave propagation model with nested solutions describing the WEC dynamics. Singh and Babarit [2,3] and Babarit et al. [4] proposed a model which solves the linear potential theory with boundary element method (BEM) around each isolated WEC, and the interactions resulting from the scattered wave field among the bodies are then taken into account via plane wave approximation in an iterative manner. McNatt et al. [5] use the commercial BEM solver WAMIT to compute the scattered wave on a circular-cylindrical section, and propose a new method to compute the cylindrical wave-field coefficients from the known cylindrical section for an arbitrary geometry. Charrayre et al. [6] proposed a methodology based on the use of a BEM model (namely Aquaplus) to solve the radiation–diffraction problem locally around each WEC, and to combine it with a model based on the mild slope equation (ARTEMIS software), within the linear wave theory. They use the Kochin function (a far field approximation) to propagate the perturbations computed by Aquaplus into Artemis.

Other approaches to achieve higher computational efficiency have been proposed in ship motion analysis related problem. Takagi et al. [7] and Ohyama and Tsuchida [8] proposed "partially 3D models", where a BEM based 3D model, applied to the near field surrounding the ship, is solved in combination with a Finite Element Method (FEM) based 2D model in the remaining sea area.

Ohyama and Tsuchida [9] derived new sets of mild slope equations for both the sea area excluding the floating body as well as for the domain underneath the body. The first set allows the reproduction of both propagating and evanescent modes, while the second set reproduces the radiated wave field. Both mild slope equations in the two domains are solved simultaneously with appropriate matching boundary conditions.

The numerical model here proposed is based on the solution of the classical linearized mild slope equation [10], at the undisturbed free surface. The near field around each floating device is excluded from the model's domain, however the hydrodynamic occurring

* Corresponding author.
E-mail address: claudia.cecioni@uniroma3.it (C. Cecioni).

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around the bodies is considered by means of appropriate novel boundary conditions. These boundaries are therefore located inside the domain and embed the near field around each floating body. The proposed model is here applied to simulate WECs of the point absorber type, and mainly it considers the hydrodynamic response of the floating devices with the incident wave field. Even if reference to such devices is made throughout this paper, the model appears suitable for other applications, which deal with hydrodynamic interaction between waves and floating bodies.

The paper is structured as follows: the next section introduces the model equations and the numerical technique. Section 3 shows the model validation, obtained by comparison with the 3D reference solution, and in Section 4 its optimization in the WECs array reproduction. Finally conclusions are given.

2. Model equations and numerical technique

The wave interaction with an array of WECs is modeled within the framework of the linearized potential theory. The wave amplitude and body motion are assumed small with respect to the wave length and body dimensions, respectively. The fluid is inviscid and incompressible, and the flow is irrotational. The bathymetry is assumed mildly sloping, i.e. $\nabla_h h \ll kh$, where $h(x, y)$ is the water depth, k is the wave number and ∇_h represents the gradient in the horizontal plane, x, y. Under these assumptions, the mild-slope equation [10] defines the space evolution for harmonic motion in terms of fluid velocity potential, $\Psi(x, y;\omega)$, at the free-surface:

$$\nabla_h \cdot (cc_g \nabla_h \Psi) + k^2 cc_g \Psi = 0 \tag{1}$$

where c and c_g are respectively the phase and group celerity. The elliptic mild-slope Eq. (1) is based on the assumption that the fluid potential can be expressed as $\Phi(x, y, z;\omega) = \Psi(x, y;\omega) f(z)$, where $f(z)$ describes the kinematic field along the water depth resulting from the linear wave theory for harmonic wave propagating in constant depth, which however still holds for mildly sloping bottom

$$f(z) = \frac{\cosh[k(h+z)]}{\cosh(kh)}. \tag{2}$$

Eqs. (1) and (2) are valid for monochromatic waves, however given the linearity of the problem an irregular sea state can be reproduced by superimposing the solutions for different wave frequencies.

As above introduced, Eq. (1) is solved over a large sea area in the presence of an array of multiple bodies. As example, in the left panel of Fig. 1 an array of 4×3 floating devices is sketched, the light gray area at $z = 0$ representing the domain of integration of Eq. (1). The thick lines represent the external open sea boundary S_O, while the dashed circular lines represent inner boundaries, S_I, where the conditions for modeling the wave field scattered by each WEC, and described in the following, are imposed. In the right panel of Fig. 1 the area around one WEC is shown. The cylindrical gray surface is highlighted to delimit the near field, around the WEC, from the far field.

The mathematical condition applied at all the boundaries, S_O and S_I, is generally formulated as:

$$\frac{\partial \Psi}{\partial n} + \alpha \Psi = G \tag{3}$$

where n is the unit vector normal to the considered boundary, α is an absorption/reflection coefficient [11], while G is a generic source term. To reproduce at the boundary S_O an open-sea condition, Eq. (3) is used with $G = 0$ and $\alpha = -ik\cos(\delta)$, i.e. δ is the angle between the wave ray and the unit vector n. In this case Eq. (3) approximates the condition of radiation of progressive planar waves [11–13]. At the same time a wave-maker condition can also be considered at S_O to simulate the entrance of incident waves into the domain; in this case the generation term G can be related to the velocity field at $z = 0$ of a given incident wave [14].

At the internal boundaries S_I, the coefficients α and G must account for the waves transmitted and scattered through the boundary by the wave-body interaction that takes place in the (not included) near field areas. By splitting each circular S_I boundary in a finite number N of arc elements, the boundary condition (3) can be imposed at each ith arc, with $i = 1, \ldots, N$, as:

$$\Psi_n^i + \alpha^{ii}\Psi^i = \sum_{j=1;\ j \neq i}^{N} \alpha^{ji}\Psi^j \tag{4}$$

where n subscript denotes the partial derivative on the normal boundary direction. Therefore, Eq. (4) relates the coefficient G^i, for the ith arc element, to the sum of fluid potential contributions from all the other arc elements on S_I boundary. The set of N equations (4) is therefore reformulated as:

$$\Psi_n^i = \sum_{j=1}^{N} \alpha^{ji}\Psi^j \quad i = 1, \ldots, N. \tag{5}$$

The computational technique to achieve the α coefficients consists on using the set of N Eqs. (5) with known values of Ψ_n and Ψ, which come from numerical or experimental test that reproduces the complete 3D wave-body interaction in the near field. In the model application presented in this paper, a numerical computation is used to solve the Laplace equation for the fluid velocity potential and the equations of the body motion. In detail, the 3D mathematical problem is described in Appendix A. Its numerical domain of application is the 3D near field around just one floating body, and its solutions at the undisturbed free-surface along the S_I boundary is used to calculate the α coefficients. If all the floating devices are identical, the α coefficients are valid for all the circular boundaries, S_I.

The system (5) of N equations contains a number of N^2 of α coefficients. The methodology to get the values of these boundary

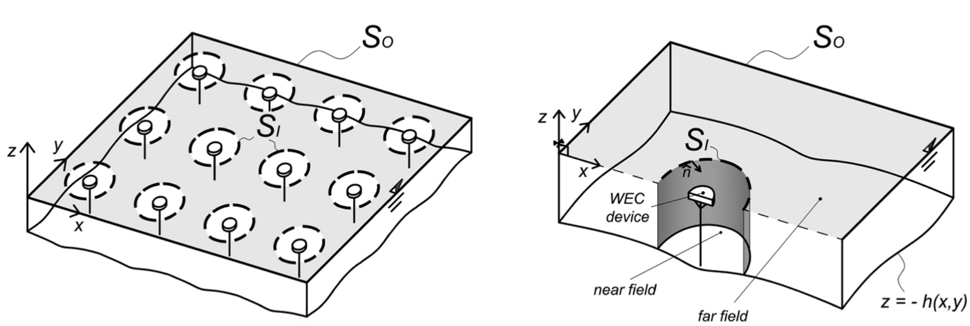

Fig. 1. Sketch of a sea area in the presence of 4×3 WECs (left plot). The gray area at $z = 0$ represents the model domain of integration. The domain's boundaries are named S_O and S_I, respectively to indicate the open-sea boundaries and the inner ones. In the right plot is shown a more restricted area, around one WEC.