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# Bragg reflection of water waves by multiple submerged semi-circular breakwaters

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ABSTRACT

This study examines the Bragg reflection of water waves by multiple submerged semi-circular breakwaters. The multipole expansions combined with the shift of polar coordinates are used to develop full linear potential solutions of the problem. In the full solutions, the obliquely and normally incident waves are independently considered. Experimental tests are carried out to measure the reflection and transmission coefficients of the breakwaters at different wave periods and body spacings. The analytical results are in reasonable agreement with the experimental data. The peak reflection coefficient of multiple submerged semi-circular breakwaters and the bandwidth of Bragg reflection are carefully examined by numerical examples. Some significant results for practical application are discussed.



## 1. Introduction

Breakwaters submerged in the sea have the merits of not affecting coastal aesthetics and allowing water circulation and fish passage. Thus, the submerged breakwaters have been often used for providing partial sheltering to coastlines and coastal structures. To enhance the sheltering function, multiple parallel breakwaters may be built. When the wave number component $k_x$ in the normal direction of the parallel breakwaters and the breakwater spacing $D$ satisfy the relation of $k_xD = n\pi$ ($n = 1, 2, \ldots$), the wave reflection can be amplified significantly, which is the so-called Bragg reflection. This study will examine the Bragg reflection of water waves by multiple submerged semi-circular breakwaters.

Bragg reflections of water waves over natural periodic sea beds or sandbars have been well studied by many researchers. Davies and Heathershaw [1] and Mei [2] examined the Bragg reflection of normally incident surface waves by periodic sandbars based on theoretical analysis and experimental tests. Kirby [3] developed a general wave equation for wave motions over rippled beds by extending Berkhoff's mild-slope equation. Mei et al. [4] studied the Bragg reflection of obliquely incident waves by sinusoidal sandbars, and found the zero reflection of bars at a critical incident angle of $\pi/4$. Guazzelli et al. [5] developed a theoretical model for the higher-order Bragg reflection of normally incident waves by periodic beds using integral matching methods, in which the slope bottom was discretized into successive steps and the eigenfunction solutions of velocity potentials in all step regions were obtained [6,7]. The integral matching methods were also used by Rey [8] and Cho and Lee [9] for analysing oblique wave reflection by either sinusoidal beds or structures. Athanassoulis and Belibassakis [10] derived a coupled-mode theory for water wave propagation over variable bathymetry regions, which could completely describe the influence of the bottom slope. Porter and Porter [11] used integral equation techniques to examine the behaviour of water waves over periodic beds.

As for artificial ocean structures, Mei et al. [4] proposed the concept of building a series of submerged bars to protect the drilling platforms in the Ekofisk of the North Sea from storm wave attack. Hence, more studies on the Bragg reflections of submerged breakwaters have been carried out. Belzons et al. [12] presented experimental evidence of the localization phenomenon for water waves over a large number of rectangular bars, which were randomly placed on the water bottom. Mattioli [13] confirmed the appearance of Bragg reflection by a series of submerged rectangular bars using matched eigenfunction expansion solution. Mattioli [13] also showed that compared to plane waves, the evanescent modes near the rectangular bars reduced the resonant frequency. Bailard et al. [14] demonstrated that a series of submerged artificial humps with suitable spacings and heights could provide storm erosion protection along U.S. Gulf Coast and Atlantic Coast beaches. Mase et al. [15] numerically and experimentally studied water wave motion over a series of trapezoidal porous bars. Their results showed that

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the Bragg reflection also occurred for multiple porous structures. Hsu et al. [16] enhanced the reflecting performance of multiple submerged rectangular bars by setting non-identical bar spacings. Cho et al. [17] compared the reflection coefficients of multiple submerged rectangular and trapezoidal bars based on calculated and experimental results, and indicated that the trapezoidal bars are better for reflecting waves. Twu and Liu [18] and Lan et al. [19] developed eigenfunction expansion solutions for Bragg reflections of water waves by rigid and elastic rectangular porous bars, respectively. Zhang et al. [20] numerically examined the porous seabed response near two and three submerged trapezoidal bars due to the Bragg reflection. Recently, Karmakar and Guedes Soares [21] developed an analytical solution for the Bragg reflection of water waves by multiple submerged porous thin barriers. Liu et al. [22] developed linear analytical solutions for long wave resonant reflections by multiple submerged bars with different shapes, including triangular, rectified cosinoidal and trapezoidal shapes. They also presented curves to determine the optimal collocations of these Bragg reflection breakwaters.

In preceding studies, the Bragg reflection breakwaters with different shapes have been carefully examined. But, the Bragg reflection of multiple semi-circular caissons has not been studied to the authors' knowledge. The first semi-circular caisson breakwater was built at Miyazaki Port, Japan in early 1990s [23]. The major merits of semi-circular caissons were summarized by Xie [24] as follows: zero overturning moment acting on the caisson as the wave pressure acting on the semi-circular arc passes through the circle centre; higher stability against sliding compared to vertical structures; and easy for building and removal. Due to these merits, the semi-circular caissons have been widely used for building breakwaters and jetties in China since 1997. In practice, the semi-circular caissons may be impermeable or permeable, and the semi-circular caisson breakwaters can be submerged or surface-piercing according to different requirements. Mallayachari and Sundar [25] developed a boundary element method solution for normally incident linear water wave motion over a single submerged semi-circular bar. Yuan and Tao [26] and Zhang et al. [27] examined the hydrodynamic performance of a single semi-circular breakwater based on numerical simulations and experimental tests, respectively. Dhinakaran et al. [28] reviewed relevant studies on semi-circular breakwaters. Liu and Li [29,30] analysed normally and obliquely incident wave actions on a single submerged permeable semi-circular breakwater using multipole expansions [31–35].

Since the semi-circular caissons have intrinsic merits as mentioned above, it may be a good idea to build multiple semi-circular bars to serve as the Bragg reflection breakwater. The present study will use analytical and experimental methods to examine the Bragg reflection of water waves by multiple submerged semi-circular breakwaters, and give some significant results for engineering applications. In the following section, the boundary-value problem of water wave scattering by multiple submerged semi-circular breakwaters is formulated. In Section 3, accurate full solutions for the present problem are developed using the multipole expansions combined with the shift of polar coordinates, where the obliquely and normally incident waves are independently considered. The experimental tests are introduced in Section 4. In Section 5, the full solutions, the wide-spacing approximations and the experimental data are compared. Numerical examples are also presented to examine the effects of major parameters on the Bragg reflection. Finally, the main conclusions of this study are drawn.

## 2. Mathematical formulation

As shown in Fig. 1, we consider water wave scattering by a series of semi-circular breakwaters (bars) submerged in the ocean with a constant depth $h$. We define a Cartesian coordinate system with the $x$-axis along the still water level and the $z$-axis pointing vertically upwards. There are total $N$ semi-circular bars, and the radius of bar $p$ is $a_p$ ($p = 1, 2, \ldots, N$). The centre of bar $p$ is located at ($x_p$, $-h$) and $x_1 = 0$. The local polar coordinate system about the centre of bar $p$ is defined with $r_p \sin\theta_p = x - x_p$ and $r_p \cos\theta_p = -(z+h)$. In the $p$-th polar coordinate system, the centre of bar $q$ ($q \neq p$) is located at ($R_{pq}$, $\alpha_{pq}$), where $R_{pq} = |x_q - x_p| > (a_q + a_p)$, $\alpha_{pq} = \pi/2$ for $q > p$, and $\alpha_{pq} = 3\pi/2$ for $q < p$. The spacing between adjacent bar centres is $D_p = x_{p+1} - x_p$ ($p = 1, 2, \ldots, N-1$). The incident waves from $x = -\infty$ propagate at an angle $\beta$ to the $x$-axis. The length of each semi-circular bar in the $y$-direction is very long compared to the wavelength and thus is assumed to be infinite.

We examine the present water wave scattering problem in the context of linear potential theory. The fluid is assumed to be inviscid and incompressible, and that its motion is irrotational. Then, we can use a velocity potential $\Phi(x, y, z, t)$ to describe the fluid motion. For time-harmonic waves with angular frequency $\omega$, the velocity potential can be further written as:

$$\Phi(x, y, z, t) = Re\left\{-\frac{\mathrm{i}gH}{2\omega}\frac{1}{\cosh(kh)}\phi(x, z)\mathrm{e}^{-\mathrm{i}k_y y}\mathrm{e}^{-\mathrm{i}\omega t}\right\}, \tag{1}$$

where $Re$ denotes the real part of argument; $\mathrm{i} = \sqrt{-1}$; $g$ is the acceleration due to gravity; $H$ is the incident wave height; $\phi(x, z)$ is a reduced spatial velocity potential; $k_y = k \sin\beta$; and $k$ is the incident wave number, which satisfies

$$K = \frac{\omega^2}{g} = k \tanh(kh). \tag{2}$$

The velocity potential $\phi(x, z)$ satisfies the modified Helmholtz equation:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} - k_y^2\phi = 0. \tag{3}$$

The velocity potential also satisfies the free surface condition, the far field radiation conditions, and the impermeable conditions on the seabed and bar surfaces:

$$\frac{\partial\phi}{\partial z} = K\phi, \quad z = 0, \tag{4}$$

$$\lim_{x\to\pm\infty}\left(\frac{\partial}{\partial x} \mp \mathrm{i}k_x\right)(\phi - \phi_I) = 0, \tag{5}$$

$$\frac{\partial\phi}{\partial z} = 0, \quad z = -h, \tag{6}$$

$$\frac{\partial\phi}{\partial r_p} = 0, \quad r_p = a_p, \quad p = 1, 2, \ldots, N, \tag{7}$$

where $k_x = k\cos\beta$; and $\phi_I$ is the velocity potential of incident waves given by

$$\phi_I = \cosh(k(z+h))\mathrm{e}^{\mathrm{i}k_x x}. \tag{8}$$

The above incident wave potential satisfies Eqs. (3)–(6).

Now, a complete boundary-value problem for wave scattering by multiple submerged semi-circular breakwaters is formulated in terms of Eqs. (3)–(8). In the following section, we will find the full solutions of velocity potentials using multipole expansions combined with the shift of polar coordinates. The obliquely and normally incident waves will be independently solved.

## 3. Multipole expansion solutions

### 3.1. Obliquely incident wave solution

Multipoles are singular solutions of governing Eq. (3) with boundary conditions in Eqs. (4)–(6). The Multipoles singular at ($x$,