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Refined seaward boundary conditions for modelling shallow water waves in semi-enclosed water bodies

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ABSTRACT

Based on the theory of characteristics, this research elaborates on the numerical treatment of two types of seaward boundary conditions for modelling long-wave dynamics in truncated estuarine and coastal domains. These seaward boundary conditions are devised for the solution of the fully non-linear shallow water equations in the time domain. The first type is the clamped boundary, at which the water level variation is given and the velocity is computed along the characteristic line going out of the domain. The second type is the non-reflecting boundary, where the incident wave information is introduced and the reflected waves from inside the computational domain are allowed to escape at the same time. The essence of its numerical implementation is to distinguish the inward and outward characteristics and to disconnect the incoming characteristic relation from the actual flow inside the domain. Compared with previous techniques, the present method includes extra terms in the derivation to account for the effects of the uneven bed, bottom friction and shape of the characteristic lines. A shock-capturing finite difference method is used to solve the shallow water equations in the deviatoric format, but the seaward boundary algorithms constructed herein are generic and applicable to other solvers. The necessity of these refinements is highlighted by simulating the tidal oscillation in the Persian/Arabian Gulf, periodic wave runup on the coastline and the wave resonance in a narrow harbour. It is found that neglecting the bed slope at the boundary may result in biased mean water levels in the prediction.

1. Introduction

When flows are confined within a thin layer, the three-dimensional incompressible Euler equations can be integrated over the depth to yield the shallow water equations. The depth-integrated equations greatly simplify the analyses, as the vertical dimension disappears and the water surface positions can be directly obtained through the conservation of mass. The shallow water equations assume the hydrostatic pressure distribution and are suitable for describing water waves with relatively long periods, so that the wavelengths are much greater than the water depths, such as tides, tsunamis and storm surges. When propagating to shallow waters, wind waves and swells can also be reasonably described with the shallow-water approximation. In all these cases, the vertical velocity and acceleration of the fluid particles are neglected. Traditionally, the shallow water equations were solved numerically using the alternating direction implicit schemes [1–3]. Recently, various shock-capturing schemes have been increasingly adopted [4–7]. Irrespective of the numerical schemes used, well-posed boundary conditions are a prerequisite for the correct solution of these partial differential equations. If proper boundary conditions are not well enforced in the discretisation, the computation over the whole domain may be jeopardised.

In the numerical study of the flows in estuarine and coastal regions, open seaward boundaries are commonly encountered, unless the "infinite" element is deployed to avoid any domain truncation [8]. The open seaward boundaries serve as the interface between the limited computational domain and the rest of the

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sea. Compared to other types of boundaries, such as wall boundaries and constant influx boundaries, the execution of the seaward boundary condition is more complicated, and thus requires closer scrutiny. For tidal oscillations inside semi-enclosed basins, such as estuaries and bays, the water level fluctuation at the seaward boundary is a major driving force, while the flows from the river mouths play only a minor role. The seaward boundaries of this type are herein called clamped seaward boundaries, where the water levels are fixed at given values. With the water levels directly prescribed outside the computational procedure, the main purpose of the boundary treatment is to calculate the appropriate velocities at the seaward boundary so that the computational stencils for all inner grid points are filled [9,10]. In simulating the wave-scattering problems, the incident wave needs to be specified at the seaward boundary, while simultaneously the outgoing waves should be allowed to freely leave the finite-sized domain through the seaward boundary. The boundaries of this type are denoted as the non-reflecting boundaries in this paper, but are also sometimes referred to as the radiation boundaries in the literature.

It is worth reiterating that the seaward boundary discussed in this paper provides the dominant forcing to the flow, without which water inside the computational domain remains largely quiescent. Although relevant, they are different from the passive open boundaries, which are sometimes termed as transparent, transmissive or radiative boundaries. The main purpose in designing the passive open boundary condition is to allow the outgoing waves to pass through the boundary with minimal reflection, whilst no key information needs to be prescribed at the passive border. The enforcement of passive open boundaries can be accomplished with the Sommerfeld radiation boundary condition or the sponge layer method [11]. In ocean modelling, Sommerfeld condition and its variants have been widely used, which can also be interpreted from the perspective of characteristics [12]. By neglecting the velocity of the fluid particle, the characteristic information is often regarded to travel at the celerity of gravity waves. The simplest variant of the Sommerfeld condition is the interpolation technique [13]. A wealth of reviews of various open boundary methodologies and comparative assessments can be found in Chapman [9], Blumberg and Kantha [14], Palma and Matano [15], McDonald [16], Nycander and Döös [17], Herzfeld [18], etc.

The present paper concentrates on the enforcement of the clamped and non-reflecting seaward boundary conditions. All of the previous methods for incorporating these boundary conditions contain certain degrees of restrictions on their usage. For instance, the effect of the bottom slope is neglected [10,13,19], empirical parameters are involved in the method [11,19], and the coupled nature of the shallow water equations is not fully considered [9]. To remove these limitations, this paper presents refinements to the characteristic-based seaward boundary conditions, which distinguish the incoming and outgoing information, and examines their effectiveness with two examples. The refinements are demonstrated to offer improved results primarily by removing the bias in the predicted mean water level.

2. Nonlinear shallow water model

2.1. Governing equations

By neglecting the baroclinic effect in the nearly-horizontal flows with free surfaces, the two-dimensional shallow water equations can be derived, which have been widely used in hydro-environmental studies. In Cartesian coordinates, these depth-integrated equations can be expressed in the following vectorised conservative form:

$$\frac{\partial \mathbf{X}}{\partial t}+\frac{\partial \mathbf{F}}{\partial x}+\frac{\partial \mathbf{G}}{\partial y}=\mathbf{S} \tag{1}$$

where t is time, x and y are the two horizontal coordinates, $\mathbf{X}$ is the conservative variable vector, $\mathbf{F}$ and $\mathbf{G}$ are the fluxes in the x and y directions respectively, $\mathbf{S}$ is the source term. For the numerical model to possess the shock-capturing capability, it should be based on this conservative form of the equations.

The vectors in Eq. (1) can be formulated as:

$$\mathbf{X}=\begin{bmatrix}\eta\\ p\\ q\end{bmatrix}, \tag{2a}$$

$$\mathbf{F}=\begin{bmatrix}p\\ pu+\dfrac{g\eta^2}{2}+gh\eta\\ pv\end{bmatrix}, \tag{2b}$$

$$\mathbf{G}=\begin{bmatrix}q\\ pv\\ qv+\dfrac{g\eta^2}{2}+gh\eta\end{bmatrix} \tag{2c}$$

$$\mathbf{S}=\begin{bmatrix}0\\ g\eta S_{bx}-gHS_{fx}+gHS_{wx}+fq\\ g\eta S_{by}-gHS_{fy}+gHS_{wy}-fp\end{bmatrix} \tag{2d}$$

where η is the water surface elevation above datum; p and q are the volumetric discharges per unit width in the x and y directions, respectively; h is the depth below the datum; $H\,(=h+\eta)$ is the total water column depth; $u\,(=p/H)$ and $v\,(=q/H)$ are the velocity components in the x and y directions respectively; $g = 9.81\ \mathrm{m/s^2}$ is the gravitational acceleration; $f\,(=2\Omega\sin\phi)$ is a coefficient for the Coriolis acceleration, with the angular speed of the Earth's rotation $\Omega = 7.29\times10^{-5}$ rad/s and the geographical angle of latitude ϕ; the bed slope, the friction slope and wind contribution are:

$$\begin{aligned}
S_{bx}&=\frac{\partial h}{\partial x} && \text{(a)}\\
S_{by}&=\frac{\partial h}{\partial y} && \text{(b)}\\
S_{fx}&=\frac{u\sqrt{u^2+v^2}}{H\cdot C^2} && \text{(c)}\\
S_{fy}&=\frac{v\sqrt{u^2+v^2}}{H\cdot C^2} && \text{(d)}\\
S_{wx}&=\frac{\rho_a C_w W_x\sqrt{W_x^2+W_y^2}}{\rho g H} && \text{(e)}\\
S_{wy}&=\frac{\rho_a C_w W_y\sqrt{W_x^2+W_y^2}}{\rho g H} && \text{(f)}
\end{aligned} \tag{3}$$

Here, C is the Chézy roughness coefficient widely used in hydraulics; ρ and ρ_a are the densities of water and air, respectively; W_x and W_y are the wind velocity components in the x and y directions, respectively; C_w is the air–water drag coefficient taken to be 0.565×10^{-3} considering that the wind velocity in this study is no more than 5 m/s.

The Chézy roughness coefficient can be linked to the Manning roughness coefficient or the bed roughness height in the following manner [3]:

$$C=\frac{1}{n}H^{1/6} \tag{4}$$