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Interaction of surface gravity wave motion with elastic bottom in three-dimensions

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ABSTRACT

A general three-dimensional hydroelastic model is developed to study the effect of elastic bottom on surface gravity wave motion in three-dimensions under the action of uniform compressive force based on linearized theory of water wave in finite water depth. The elastic bottom bed is modeled as a thin plate theory. The progressive wave characteristics in different wave modes are analyzed in both the cases of deep and shallow water waves. Further, the linearized long wave equation under shallow water approximation in a direct manner is derived and compared the results obtained based on the small amplitude wave theory. Three-dimensional Green's function associated with surface gravity wave motion with elastic bed is derived using the fundamental source potentials. Fourier-type expansion formula and corresponding orthogonal mode-coupling relations are derived in finite depth and finite/semi-infinite width in three-dimensions. Utilizing the expansion formula, two classical problems (i) forced motion and (ii) wave reflection by a rigid wall in a channel of finite width and finite/semi-infinite length are illustrated which will play significant role in the analysis of wave-structure interaction problems arising in ocean engineering.

The effects of elastic bottom on surface gravity waves are analyzed by presenting several numerical results on reflection wave amplitudes in different cases. Further, the behavior of free motions and oscillations in a basin of finite width and finite/semi-infinite length with elastic bed under the action of uniform compressive force are discussed.

1. Introduction

In the last two decades, there is a wide interest to study in the hydroelastic behavior of large floating/submerged structures for appropriate design and construction of renewable energy floating platform, floating airports, floating bridge, floating offshore base to utilize the ocean space for various human activities and developments. Such huge structures are modeled based on thin elastic plate theory due to its advantageous in theoretical analysis and as well as in computation which has been widely used on wave-structure/ice sheet interaction problems (see [1–10]).

The mathematical complexity associated with surface gravity wave interaction with floating or submerged flexible structures is the existence of higher order boundary conditions on the structural boundaries and satisfies Laplace/Helmholtz equation as the governing equation. These problems are of non-Sturm Liouville type in nature and associated eigenfunctions are not orthogonal in usual sense.

In recent times, efforts have been made to study the various aspects of wave interaction with floating or submerged flexible structures in three-dimensions where the ocean bed is assumed to be rigid. Lawrie [11] reviewed a class of orthogonality relations relevant to fluid–structure interaction in both the cases of single and two-layer fluids. Lawrie [12] derived the orthogonality relations for rectangular duct with flexible wall in three-dimensions. Further, Lawrie [13] analyzed on the acoustic wave propagation in three-dimensional rectangular ducts with flexible wall and porous linings. Mondal et al. [14] studied the various characteristics of vertical eigenfunctions associated with wave-structure interaction in three-dimensions in single layer fluid in both the cases of finite and infinite water depths. Mohapatra et al. [15] analyzed the effect of compressive force on the wave diffraction by a floating elastic plate for different water depths

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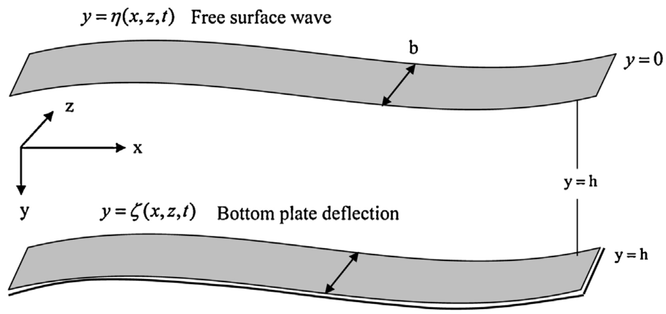

Fig. 1. Schematic diagram.

based on integro-differential equation method. Green's function associated with surface wave motion in a two-layer fluid having either a free surface or in the presence of surface tensions and interface or having interfacial tension are derived by using fundamental source potentials [16,17] which mathematical analysis are analogous to wave–structure interaction problems.

In the other hand, the elasticity of the bottom bed is also one very important aspect of study which plays an important role in various branches of engineering and applied disciplines for example, in particular when oil is loading and uploading in large-capacity oil storage tank can survive earthquakes. Some of the previous studies involving the elastic/flexible/membrane bottom effect are investigated. The free vibration analysis of the coupled-system have been studied by number of investigators (see [18–22]), by considering the flexible bottom as an elastic plate or as a membrane. Mallard and Dalrymple [23] studied the water waves propagating over a deformable bottom. Dawson [24] investigated the wave propagation over a deformable sea floor. Mohapatra and Sahoo [25] developed a hydroelastic model to study the surface wave interaction with elastic bottom in two-dimensions. Eyov et al. [26] presented a mathematical solution for two-dimensional linear problem of acoustic gravity waves in a compressible ocean with an elastic bottom to compare the physical properties of progressive waves over rigid and slowly-varying bathymetry. Saha and Bora [27] studied the effect of elastic bottom on trapped waves in two layer fluids in finite water depth by using multipole expansion method.

Although there has been a little progress in the literature on the effect of the elastic sea bed in two-dimensions, the investigations of the effect of elastic bottom on surface gravity waves in three-dimensions has not taken place to the best of the knowledge of the authors till date. Therefore, in the present study, a general mathematical model is developed to study the effect of elastic bottom under the action of uniform compressive force and finite width of the elastic bed on surface gravity wave motion in three-dimensions. The progressive wave characteristics in surface mode (SM) due to free surface and flexural mode (FM) because of elastic bed in the presence of compression are studied by analyzing the phase and group velocities associated with the linearized dispersion relation in both the cases of deep and shallow water depths (as in [25]). The linearized long wave equation under shallow water approximation in a direct manner is derived in three-dimensions. Using fundamental source potential, three-dimensional Green's function associated with surface gravity wave motion over elastic bottom is derived. Further, the Fourier-type expansion formula for velocity potentials and corresponding orthogonal mode-coupling relations are derived in three-dimensions. The application of the expansion formula is analyzed by discussing two classical problems (i) forced motion and (ii) wave reflection by a rigid wall in a channel of finite width and finite/semi-infinite in finite water depth with elastic bed under the action of uniform compressive force. The effects of elastic bottom on uniform compressive force, channel width, rigidity of the bottom plate, and modes of oscillation are studied by analyzing several numerical visual wave plots on reflection wave amplitudes in different cases. Further, the free oscillations in a basin of finite width and finite/semi-infinite length with elastic bed under uniform compressive force are discussed in three-dimensions in specific cases. The general expansion formula for velocity potential and corresponding orthogonal mode-coupling relation associated with the boundary value problems (BVP) is derived in three-dimensions in Appendix A.

2. Mathematical formulation

In the present formulation, the problem is considered in three-dimensional Cartesian co-ordinate system with x-axis being in the horizontal direction and y-axis being in the vertically downward positive direction and z is along width. The fluid domain is considered as an infinitely extended channel of finite and infinite width. The fluid region occupies $0<x<\infty$, $0<y<h$, $0\le z\le b$ (as in Fig. 1) with the free surface elevation $\eta(x, z, t)$ and the vertical displacement $\zeta(x, z, t)$ of the bottom elastic bottom plate are assumed to be of the form $\eta(x, z, t)=Re\{\eta(x, z)e^{-i\omega t}\}$ and $\zeta(x, z, t)=Re\{\zeta(x, z)e^{-i\omega t}\}$ respectively, where Re denotes the real part. Assuming that the fluid is inviscid and incompressible, the motion is irrotational and simple harmonic time with angular frequency ω. Thus, there exists a velocity potential $\Phi(x, y, z, t)$ of the form $\Phi(x, y, z, t)=Re\{\phi(x, y, z)e^{-i\omega t}\}$ where $\phi(x, y, z)$ is the spatial velocity potential without time harmonic, which satisfy the three-dimensional Laplace equation as given by

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=0 \quad \text{in the fluid domain.} \tag{1}$$

The linearized boundary condition at the mean free surface is given by

$$\frac{\partial^2\Phi}{\partial t^2}-g\frac{\partial\Phi}{\partial y}=0 \quad \text{on} \quad y=0. \tag{2}$$