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Modal response of free spanning pipelines based on dimensional analysis

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ABSTRACT

The paper examines the problem of an axially loaded pipeline resting on semi-infinite elastic supports on either side of a free span. A dimensional analysis of the problem is carried out, and it is demonstrated that the eigenfrequencies and associated maximum modal stresses of the pipe may be uniquely represented as functions of only two non-dimensional parameters. Explicit analytical formulae for the modal response quantities are derived, with improved accuracy and applicability range as compared to approximate expressions that are widely used in the pipeline industry today.

1. Introduction

As easily accessible oil supplies onshore and in shallower waters are diminishing, persistently high oil prices are driving oil and gas exploration and development into increasingly deeper waters and harsher environments [1,2]. As the water depths increase and the well conditions become more challenging, costs of seabed preparation and intervention increase accordingly [2,3]. Extremely long, large diameter gas trunklines have been and are being installed in Europe, Asia and Australia with examples such as Langeled [4,5], South Stream [6], Nord Stream [7] and Gorgon Jansz [8]. It has been estimated that more than 5000 km of intermediate and large diameter offshore pipelines are installed each year [9].

A significant challenge for long pipelines laid on rough seabed topography is so called free spans, sections of the pipeline where it does not touch down on the seabed. Unacceptable free spans may require costly seabed intervention work, and the importance of free span design is evident in leading offshore pipeline design codes such as DNV-OS-F101 [10] and API RP 1111 [11]. Both codes recommend that pipeline free span design is performed according to DNV-RP-F105 [12], which was extensively revised in 2006 specifically due to considerable challenges with free spans on the Ormen Lange pipelines [13–15]. A pipeline free span may pose a risk to pipeline integrity for three reasons. Firstly, the static weight of the pipeline may cause excessive bending, potentially causing local buckling [10,16]. Secondly, a free span makes the pipe more susceptible to being hooked by anchors or trawling equipment [17]. Thirdly, a free span allows the pipeline to respond dynamically due to vortex-induced vibrations (VIV) or direct wave loading [18,19], resulting in accumulation of fatigue damage. When not appropriately considered in design, failures have been known to occur [15].

The present paper will focus on the dynamic pipeline free span response, which in conventional pipeline design is assessed in the frequency plane [12]. Hence, modal frequencies and associated modal stresses are essential parameters. A simplified analytical modal analysis approach is detailed in DNV-RP-F105 [12], with a limited range of validity. Current industry practice for advanced scenarios (e.g., long spans or multi-spans) is to perform detailed finite element analyses [20–23]. Long pipelines may, however, have hundreds or even thousands of free spans, implying that detailed finite element (FE) analyses of all free spans may be costly and impractical. Also, in early stages of design such as feasibility studies or routing design, simple and robust free span criteria are easy to establish given simplified analytical approaches for the modal response.

Several studies have been performed aiming to establish reliable analytical methods for prediction of in-line modal response in free spans [24–30]. Simple analytical equations for beams with idealized boundary conditions, such as pinned–fixed ends, were commonly applied historically [25,31]. Kaye et al. [28] proposed fixed–fixed end conditions with an effective span length

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corresponding to 1.1 times the actual span length. Choi [24] studied the influence of axial forces and derived improved frequency expressions based on the Rayleigh method for idealized boundary conditions. Equations for natural frequencies, accounting for axial forces, were also recently suggested by Xiao and Zhao [30] based on regression to FE analyses with pinned–fixed ends. However, the importance of considering the stiffness of the span shoulders was clearly demonstrated by Hobbs [26], who solved the governing differential equation for a free spanning beam resting on an elastic foundation. Hobbs did not, however, derive closed-form equations for the modal response or include axial forces. Park and Kim [29] modeled span shoulders using linear and rotational springs and provided dimensionless free span response curves, disregarding axial forces. Elastic shoulders were included in a similar manner by Kapuria et al. [27], who also considered axial forces. Numerical results were presented as non-dimensional curves without providing closed form analytical expressions [27]. None of the above-mentioned studies considered modal bending stresses.

Fyrileiv and Mørk [25] derived a semi-empirical solution, based on work done by Hetenyi [32] and Hobbs [26] as well as calibration to a large number of FE analyses, and proposed simple analytical expressions for eigenfrequencies and associated modal stresses. Their frequency equation accounts for soil stiffness and axial forces, and was shown to be in good agreement with the results of FE analyses within a limited validity range. The expressions developed by Fyrileiv and Mørk [25] have been adopted by DNV in their recommendations and implemented in the DNV code compliance software FatFree, and are widely used by the pipeline engineering community [12,33].

In the present study, new modal response calculation methodologies for pipeline free spans are developed. The accuracy and range of validity compared to previous formulations are significantly improved, while still maintaining closed analytical solutions which are robust, transparent and easy to use. Furthermore, a complete solution to the governing differential equation for a free spanning, axially loaded pipeline resting on elastic shoulders is presented (for the first time, to the authors' knowledge), and a transcendental equation for determining the exact eigenfrequencies is established. A dimensional analysis is performed based on the Buckingham Pi theorem, which allows for visualization of response surface solutions to both frequencies and modal stresses. The proposed modal response expressions are based on surface fitting to the non-dimensional response surfaces and also expand the effective length concept introduced by Hobbs [26] and Fyrileiv and Mørk [25]. The accuracy of the novel closed analytical expressions is thoroughly investigated and improvements compared to the existing formulation in DNV-RP-F105 are identified. The study is restricted to the response of single spans in the horizontal direction, i.e., effects of sagging due to self-weight and potential interaction with neighboring spans are not considered.

2. Problem definition

2.1. Physical description

A typical pipeline cross-section includes a Carbon Manganese (CMn) high strength steel, sometimes with an internal corrosion resistant liner [34,35] and various types of external coatings for corrosion protection, thermal insulation [36] or weight [37]. Pipeline joints are connected by circumferential (girth) welds at each end (typically 40 ft/12.2 m), and each connection is called a field joint.

Free spans occur due to seabed topography, scouring or interference from other installations. The length of pipe suspended above the seabed is called the span length, L_s, and areas near the span where the pipe touches down on the seabed are called span shoulders. A picture of a concrete weight coated pipe in a free span is shown in Fig. 1.

Fig. 1. A free spanning concrete coated pipeline (photo: DNV GL).

In free span design, the fatigue damage resistance is normally assumed equal to the resistance of the girth weld, since this is much lower than the fatigue resistance of the pipe steel [12]. Since pipelines are submerged in water, static and dynamic response calculations must account for hydrodynamic added mass and external pressure. The total dry mass m_d of a pipeline includes the pipe steel and all coating layers. In dynamic analyses, the total effective mass m_e of the pipe includes the dry mass, the content mass and the added mass. When a pipe bends, the internal and external pressures cause net transverse loading directed along the radius of curvature [38,39]. An efficient means of accounting for transverse loading caused by pressures is to replace the pipe wall axial force N with the effective axial force S_{eff}, defined by Eq. (1):

$$S_{eff} = N + p_e A_e - p_i A_i, \tag{1}$$

where p_e and p_i are the external and internal hydrostatic pressures, respectively, and A_e and A_i are the external and internal cross-sectional areas. The axial forces are defined as positive in tension.

2.2. Mathematical model

Two free span models will be applied. These are illustrated in Fig. 2. For the novel analytical solutions, infinitely long shoulders are modeled as elastic foundations with lateral soil stiffness K_L. In the semi-analytical model, which is used to generate response surfaces and to study the accuracy of the novel analytical solutions, span shoulders of finite length $L_{shoulder}$ are applied.

Shear deformations have a negligible impact on the modal response of offshore pipelines [12,22,40], and thus Euler–Bernoulli beam theory is considered applicable for modeling of the pipeline. The differential equation for a freely vibrating Euler–Bernoulli beam subjected to axial force is given, e.g., by Clough and Penzien [41]. After including restoring forces from the soil, it becomes

$$EI\frac{\partial^4 v}{\partial x^4} - S_{eff}\frac{\partial^2 v}{\partial x^2} + k_y v + m_e\frac{\partial^2 v}{\partial t^2} = 0, \tag{2}$$

where E is the Young's modulus and I the area moment of inertia. The soil stiffness $k_y = K_L$ on the shoulders and zero in the span. Eq. (2) is conventionally solved by separation of variables, which yields

$$\ddot{Y} + \omega^2 Y = 0, \tag{3}$$

$$\frac{d^4\phi}{dx^4} - \frac{S_{eff}}{EI}\frac{d^2\phi}{dx^2} + \left(\frac{k_y}{EI} - \frac{m_e\omega^2}{EI}\right)\phi = 0, \tag{4}$$