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Linearised water wave problems involving submerged horizontal plates

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ABSTRACT

In this paper a number of related linearised water wave problems all involving thin submerged horizontal plates are considered. An integral transform approach is adopted and used to formulate integral equations in terms of unknown functions related to the jump in pressure across the plate. A Galerkin method is applied to the solution of these integral equations leading to elegant expressions for quantities of interest and a rapidly convergent numerical scheme. The focus of the paper is to demonstrate the application of this method in a number of settings including both two-dimensional problems applied to infinitely-long plates of constant width and three-dimensional problems involving circular discs. In the process we present new results including, for example, for wave-free forced oscillations of plates.

1. Introduction

In this paper a number of problems relating to the interactions of surface gravity waves with thin horizontal plates are considered under the assumptions of linearised wave theory. These include: (i) the scattering of oblique waves by a plate of constant width; (ii) radiation from forced motion of plates and (iii) the scattering of waves by a circular plate. The main purpose of the paper is to demonstrate a new approach to solving a general class of problem involving thin submerged horizontal plates which has some advantages over existing solution methods.

The interaction of surface waves with submerged thin horizontal plates has been the subject of numerous studies over the past decades partly due to its potential application as either a submerged breakwater or underwater lens and partly because of an intrinsic interest in methods for solving boundary-value problems involving thin structures. Alongside numerical and experimental investigations, many different analytical solution methods and approximations have been developed. Some of the studies relevant to the current work will be listed below; many others are cited within these references.

Some solution methods are specific to the water depth being finite. For example, the popular eigenfunction matching method divides the fluid domain into four rectangular subdomains: above and below the plate and to the left and right of the plate. Expanding the velocity potential in each subdomain in terms of separation solutions and then matching velocities and pressure across common vertical boundaries leads to infinite systems of linear algebraic equations which can solved numerically by truncation. See, for example, McIver [1] or Mahmood-ul-Hassan et al. [2]. As is often the case with so-called eigenfunction matching method, this approach is numerically intensive, requiring large series truncation sizes for modest levels of accuracy. This can be attributed to the fact that the method does not explicitly take account of an essential requirement of the problem relating to the behaviour of the fluid at special points within fluid domain, in this case at the edges of the thin plates.

A variant of the eigenfunction matching method (in that it is based on the same domain decomposition) is the modified residue calculus approach (for a general description see Linton and McIver [3]) which *does* account for the specific behaviour at the edges of the plate. This sophisticated method gives identical results to those found from the Wiener–Hopf technique when applied to the problem of a semi-infinite plate where solutions are explicit (e.g. Greene and Heins [4], Heins [5]); this equivalence is explicitly demonstrated in Williams and Meylan [6].

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For plates of finite length, Fernyhough [7] and Linton [8] showed that the residue calculus method results in systems of equations that are exponentially convergent. Additionally, when truncated to leading order, they are equivalent to a wide-spacing result which connects the scattering effects from the two edges of the plate using propagating waves only, assuming each edge is characterised by the scattering from a semi-infinite plate.

Alternative approaches include using Green's functions either in finite or infinite depth (see, for example, Linton and McIver [3]) where certain types of integral equation can be formulated. The main difficulty lies in how the integral equations are solved, and this is expanded upon further below.

There are also a number of approximations that are used. The wide-spacing approximation referred to above (e.g. McIver [1]), Linton and McIver [3]) is accurate when the wavelength over the plate is much less than the length of the plate. The shallow water approximation of Siew and Hurley [9] based on matched asymptotics accurately captures the solution in the long wavelength limit.

The method of solution presented here involves the use of either Fourier or Hankel transforms to formulate integral equations for functions related to the unknown jump in pressure across the plate. It is natural to consider transform methods when considering problems which possess coordinate-aligned geometry as is present here. The solution is approximated accurately and efficiently by use of a variational approach (Galerkin's method) in which the unknown is expanded in an orthogonal basis which incorporates the anticipated square-root behaviour in the pressure jump at the edges of the plate. Quantities of interest in the problem are readily expressed in terms of inner products involving the solutions to the integral equations and, consequently, are second order accurate due to the variational approximation adopted.

Perhaps unsurprisingly, there are a number of similarities in the approach presented here and methods based upon Green's functions referred to above. The present approach sets itself apart from previous work primarily in the way various equations are organised. The integral equation that is derived in this paper resulting from a transform approach is equivalent to the integral equation resulting from the use of a Green's function once its integral transform representation had been used. The introduction of a transform variable, either from the outset or, later, in a Green's function representation, is crucial to the present development and the ordering of domain and transform integrals allow the integral equation to be handled 'normally'. If the order of domain and transform integrals are reversed, the integral equations become hypersingular and immediately requires special attention. Hypersingular integral equations emerge naturally in the application of Green's functions to thin plates and can be dealt with by developing the appropriate machinery (e.g. Parsons and Martin [10,11], Farina and Martin [12]) or by using 'regularisation methods' which integrate away the hypersingularity by switching normal to tangential derivatives to leave weakly singular integral equations (e.g. Porter [13]). This latter approach has some common features with the work of Grue and Palm [14] and Song and Faltinsen [15], for example, who use distributions of point vortices around submerged plates to formulate non-singular integral equations which are solved numerically after expanding the unknown vortex strength in terms of Fourier series. Many of the Green's function approaches described above have the advantage that they have been applied more generally to geometries where transforms are not appropriate. However, the main point of the present work is to demonstrate that this machinery is not necessary for the class of problem considered here.

The basic method being applied here in the context of water waves is known in other branches of scattering theory. See, for example, the review paper by Boström [16] and the discussion contained within the appendix of the recent paper by Farina et al. [17] on connection to methods based on dual integral equations.

A by-product of the current approach is the development of an explicit long wavelength approximation to wave scattering by submerged horizontal plates, formally valid in $\kappa a \ll \pi$ where κ will come to represent the wavenumber of travelling waves over the plate. This new approximation complements existing approximations which include the long-plate (or wide-spacing) approximation, formally valid for $\kappa a \gg \pi$ and the shallow-water approximation. The performance of these existing approximations is described in McIver [1] and is not a focus of attention here.

In Section 2, the scattering of waves by a fixed plate is considered in finite depth. The case of infinite depth is also found explicitly by letting the depth tend to infinity. Results are shown for reflection and transmission coefficients, demonstrating the rapid convergence of the numerical approximation with increasing number of terms in the series. In Section 3 the modification to the formulation is presented in the case of radiation of waves by the forced motion of the plate. Here we demonstrate that plates in heave or roll motions about the plate centre can produce local wave-free oscillations, and that off-centre rolling plates can radiate waves in one direction only. In Section 4 an integral equation formulation is derived for the three-dimensional extension of the plate to a submerged horizontal circular disc. There are some differences, particularly in the numerical solution method although the final numerical systems of equations are remarkably similar to that required in the two-dimensional problem.

2. Scattering of oblique waves by a submerged plate of constant width

Cartesian coordinates are used with $z = 0$ in the mean free surface and the fluid extending into $z < 0$. A thin horizontal plate is submerged to a depth d in water of depth h. The plate extends horizontally from $x = -a$ to $x = a$ and uniformly in the y direction. Assuming time-harmonic incident waves of angular frequency ω making an angle θ with respect to the positive x direction, the governing equation to be satisfied by the velocity potential $\phi(x, z)$ is

$$(\nabla^2 - \beta^2)\phi(x, z) = 0, \quad z < 0 \tag{1}$$

where $\beta = k \sin\theta$ where k is the positive root of the dispersion relation

$$\frac{\omega^2}{g} \equiv K = k \tanh kh. \tag{2}$$

The velocity field is reconstructed from the gradient of $\Re\{\phi(x, z)e^{i(\beta y - \omega t)}\}$. In addition to (1), on the free surface

$$\phi_z - K\phi = 0, \quad z = 0 \tag{3}$$

and on the lower boundary of the fluid,

$$\phi_z = 0, \quad z = -h. \tag{4}$$