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Optimisation of arrays of flap-type oscillating wave surge converters

I.F. Noad*, R. Porter

School of Mathematics, University Walk, Bristol BS8 1TW, UK

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ABSTRACT

In this paper a finite array of hinged flap-type wave energy converters are modelled using a mathematical approach. These are illustrative of the Oyster device of Aquamarine Power Ltd.[1] A novel semi-analytic solution method is presented for a set of boundary-value problems involving the scattering and radiation of waves by thin barriers used to model the device hydrodynamics. The approach makes use of the geometry to apply Fourier transforms, deriving non-singular integral equations in terms of the jumps in pressure across the flaps. These are then solved numerically using a highly efficient Galerkin expansion method. The focus of the results is on optimisation. We suggest optimal parameters for a single device, identifying flap length as crucial to device performance. This optimisation is then carried through to arrays with optimal arrangements and spacings being determined for a model sea state. Here, the lateral displacement of the devices emerges as a critical factor in optimal array configuration.

1. Introduction

The development of technology for the conversion of ocean wave energy into a source of useable energy has been a long and slow process. Interest in the potential of wave energy converters (WECs) as a significant source of renewable energy started in earnest in the mid-1970s. However, the route to full scale commercial deployment has been plagued by a combination of factors, not least the difficulty in engineering devices exposed to the harsh marine climate which are both reliable and efficient. In the UK there are two devices which have recently emerged as promising candidates for large scale commercial success: the Pelamis device manufactured by Ocean Power Delivery and the Oyster device of Aquamarine Power Ltd.[1] Although engineering development challenges remain both have enjoyed some success as single device prototypes and the emerging challenge is in extending to multiple devices in a wave farm array.

This paper uses a mathematical approach to consider a variety of aspects concerned with the operation of a hinged flap-type device which is illustrative of the Oyster device referred to above. The Oyster device itself was derived from research carried out at Queens University, Belfast, UK (see [21], for example), and we follow some of the basic modelling assumptions used in that early work. Thus, we consider a wave energy converter that is assumed to operate in shallow water environments. This is comprised of a long, buoyant, rectangular paddle extending upwards through the water surface. The paddle is hinged along a horizontal axis on a fixed foundation protruding vertically from the sea bed. When waves are incident upon the flap, it pitches about the submerged hinge and power is generated by this pitching motion relative to the fixed foundation. In the hydrodynamic modelling needed for the computation of power output we make a number of assumptions. The first is that the thickness of the flap-type paddle and its foundation are small enough with respect to typical wavelengths to be regarded as infinitely thin. The next is to assume that linearised wave theory can be employed, an approximation requiring wave steepnesses and paddle pitching angles to be small enough. Such approximations are standard in the analysis of wave energy devices, see Cruz [5] for example.

To date, much of the development work on Oyster has been carried out using numerical CFD and experimental wave tank testing [21,9]. However, recently a series of papers, [15–17] have approached the hinged flap-type problem. Thus, an infinite periodic array, a single device and a finite number of in-line devices have all been considered. Our paper uses the same background theory as contained in [15,16]. However, there are some differences which will be alluded to in the description of our approach below.

The main purposes of the present paper are twofold. First to demonstrate a mathematical solution method to the hydrodynamic problems that arise when considering a finite array of N flap-type devices which are parallel but otherwise positioned arbitrarily. Thus it is shown how an application of Fourier transforms leads to N coupled integral equations in terms of N unknown functions relating to jumps in hydrodynamic pressures across the flaps. Furthermore, application of a Galerkin approximation involving a judicious choice of expansion functions reduces the integral

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* Corresponding author. Tel.: +44 7892 829 177.
E-mail addresses: imogen.noad@bristol.ac.uk (I.F. Noad), richard.porter@bristol.ac.uk (R. Porter).

[1] http://www.aquamarinepower.com/.

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equations to a low-order system of equations whose solutions are efficiently and accurately computed. This approach is different to Renzi and Dias [16], Renzi et al. [15] who used Green's functions to develop hypersingular integral equations, solved by collocation.

The second purpose is to exploit the numerical efficiency of solutions for arrays of devices to perform an optimisation over a number of free parameters associated with the theoretical problem. This allows us, in the first instance, to assess the optimal configuration of a single flap device under a realistic random sea state. In particular, it is demonstrated that the length of the flap is critical to its performance. Continuing further, the configuration of a multi-flap array is considered, the optimisation procedure being used to select the arrangement of the array which yields the highest total power output, again in a model sea state. Here, the stagger and distances between elements of the array emerge as critical factors in determining array performance.

One of the difficulties in designing arrays of devices – and the reason why we have resorted to using numerical optimisation in this paper – is that there are limited theoretical results for optimisation of power from multiple elements of an array. This is in stark contrast to what is known about how single devices work and are optimised. In Appendix A we have provided a series of results relating to optimal power for arrays of devices under practical constraints on the power take-off. It is hard to imagine that all of these results are new, although the authors have been unable to find them in the literature – perhaps because they have limited application. However, for an array of two identical devices new results of practical use have been derived for the maximisation of power.

In Section 2 of the paper we derive expressions for the power absorption and relate them to certain properties of scattering and radiation potentials satisfying linear water wave problems and associated with the scattering and radiation of waves by each of the thin barriers. Much of this section is guided by principles of wave power conversion calculations set out, for example, in Thomas [19]. These are then extended in Appendix A where strategies are developed for determining optimal mechanical damping for practical power take-off systems in the context of an array and new results are derived. We go on to specify the hydrodynamic problems associated with the scattering and radiation of waves by a finite array of parallel flaps in Sections 3 and 4 a new integral equation formulation is derived and presented for their approximate solution.

Certain elements which are key to the numerical calculations and accuracy of the subsequent approximate solutions are discussed in Section 5. It is here that we introduce a self-similar spectrum applicable to the near-shore context and used in the determination of the mean capture factor. This is employed throughout the results as a measure for optimal performance in random waves representative of a real sea. Results are then presented in Section 6; initially for a single device and then, with increasing generality, for arrays of 2, 3 and 5 devices. Finally, in Section 7 conclusions are drawn and suggestions for future work are given.

In the final preparation for submission of this paper the authors became aware of a recently published paper, Sarkar et al. [18], which considers the same problem of power take-off from arbitrary arrays of parallel flap-type devices. There are, of course, inevitable similarities between the paper of Sarkar et al. [18] and the present paper. Thus, the hydrodynamical modelling and model assumptions are the same and the focus on assessing the performance differences between in-line and staggered arrays is similar. Some of the conclusions are similar too. However, there are some significant differences in the two pieces of work. The features of our paper highlighted earlier in Section 1, such as the mathematical analysis of the single device performance and the development of analytic expressions for optimising power for arrays, are new. In terms of the approach taken to solve the hydrodynamic problem, Sarkar et al. [18] use Green's functions to develop hypersingular integral equations numerically approximated by collocation methods. In contrast, we have taken a very different mathematical approach, using Fourier transforms to develop non-singular integral equations which are numerically approximated by Galerkin's method. This difference is significant in terms of numerical efficiency of computations. Numerical simulations on a test problem performed by Sarkar et al. [18] on a 3.4 GHz PC with 16 GB of RAM are quoted as taking 6 min on average (page 7). On a similar piece of equipment (a 3.0 GHz PC with 16 GB RAM) our method applied to their same problem is roughly 20 times faster. This increase in numerical efficiency is crucial if one wants to implement optimisation methods as we have done here.

Finally, the focus of the numerical results is very different here to Sarkar et al. [18] where the main focus is on q-factors for central device elements as functions of wave period. There, discrete configurations of elements in the array have been considered with fixed spacings and mainly in normally incident waves. We have instead focused on the total power developed by the whole array under a random wave spectrum (adjusted for the nearshore environment) with directional spreading and, instead of fixed spacings, we have implemented an optimisation routine which selects the optimal array configuration. Some of the conclusions made by Sarkar et al. [18] we agree with, such as general rules preferring certain types of stagger over others. However, the results presented here have been able to select optimal configurations at spacings well beyond those considered in Sarkar et al. [18].

2. Formulation

In formulating the problem for an array of N devices we exploit the decomposition ideas recently presented by Renzi and Dias [16] which were applied to the related problem of a single device. Cartesian coordinates have been chosen with the origin at the mean free surface level and z pointing vertically upwards. The fluid has density ρ and is of constant, finite depth h. We consider a finite array of N flap-type devices, each labelled by the index n. These are oriented parallel to the y-axis, have length denoted by $2a_n$ and are centred (when viewed from above) at the points $(x, y) = (b_n, d_n)$ such that $b_1 \le b_2 \le \cdots \le b_N$. The hydrodynamic model assumes the flaps are infinitely thin and buoyant so that when at rest they occupy the vertical planes $\{x = b_n, d_n - a_n < y < d_n + a_n, -h < z < 0\}$ for $n = 1, \ldots, N$. They are hinged along horizontal axes $(x, z) = (b_n, -c)$, which are denoted in Fig. 1 by P. Above its pivot each flap is free to move and below it is held fixed and vertical. $\Theta_n(t)$ is the (assumed small) angle through which the nth flap has rotated measured anticlockwise from the vertical. A standard small-amplitude, linearised theory of water waves is used. A monochromatic wave of assumed small amplitude $H/2$ and radian frequency ω is incident on the array from $x < 0$ making an anti-clockwise angle β with the positive x-direction where $\beta \in (-\pi/2, \pi/2)$.

The time-dependent problem is as follows. We define a velocity potential $\Phi(x, y, z, t)$ satisfying

$$\nabla^2 \Phi = 0, \quad \text{in the fluid} \tag{2.1}$$

with linearised dynamic and kinematic free surface conditions

$$\Phi_t + g\zeta = 0, \quad \text{and} \quad \zeta_t = \Phi_z, \quad \text{on } z = 0 \tag{2.2}$$

where $\zeta(x, y, t)$ denotes the free surface and g is the gravitational acceleration. On the bottom of the fluid,

$$\Phi_z = 0, \quad \text{on } z = -h. \tag{2.3}$$