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Near-optimal control of a wave energy device in irregular waves with deterministic-model driven incident wave prediction

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ABSTRACT

This paper investigates wave-by-wave control of a wave energy converter using incident wave prediction based on up-wave surface elevation measurement. The goal of control is to approach the hydrodynamically optimum velocity leading to optimum power absorption. This work aims to study the gains in energy conversion from a deterministic wave propagation model that accounts for a range of group velocities in deriving the prediction. The up-wave measurement distance is assumed to be small enough to allow a deterministic propagation model, and further, both wave propagation and device response are assumed to be linear. For deep water conditions and long-crested waves, the propagation process is also described using an impulse response function (e.g. [1]). Approximate low and high frequency limits for realistic band-limited spectra are used to compute the corresponding group velocity limits. The prediction time into the future is based on the device impulse response function needed for the evaluation of the control force. The up-wave distance and the duration of measurement are then determined using the group velocity limits above.

A 2-body axisymmetric heaving device is considered, for which power capture is through the relative heave oscillation between the two co-axial bodies. The power take-off is assumed to be linear and ideal as well as capable of applying the necessary resistive and reactive load components on the relative heave oscillation. The predicted wave profile is used along with device impulse response functions to compute the actuator force components at each instant. Calculations are carried out in irregular waves generated using a number of uni-modal wave spectra over a range of energy periods and significant wave heights. Results are compared with previous studies based on the use of instantaneous up-wave wave-profile measurements, both without and with oscillation constraints imposed. Considerable improvements in power capture are observed with the present approach over the range of wave conditions studied.

1. Introduction

Wave energy conversion has received considerable attention in the literature for over four decades now [2–4]. A large number of wave energy converters studied so far have consisted of a wave-activated body and a reference, for energy conversion from the relative oscillation. Since power absorption is best close to resonance and wave climates can vary widely through a year, approaches to provide active control of device oscillations have also been pursued for many years. Early efforts to increase response bandwidth by controlling the phase of the force applied by the power take-off were reported in [5,6]. This control was accomplished by including a reactive component in the load in addition to the resistive part and independently adjusting the magnitudes of the two parts. Further investigations on small heaving point-absorber devices with short resonant periods led to the development of the 'latching' concept [7]. This improved power capture in longer-period waves by keeping body oscillation locked and releasing it only when the velocity peak was close to matching the exciting force peak.

The decades since the seventies have witnessed a growing realization that some type of control may be essential for cost-effective implementation of wave energy converters (e.g. [8]). Thus, latching type switching control has been investigated by many researchers to date (e.g. see [9–14], etc, to name a few). The determination of an optimal latching sequence was first addressed under an optimal control framework in [9] with the dynamic model providing the dynamic constraints. The optimum switching sequence can be determined with the help of the Pontryagin Max/Min Principle (often following an iterative approach). Although latching requires no reactive load, the co-state equations describing the time-dependence of the Lagrange multipliers need to be integrated back in time from T_f to 0 in order to determine a latching sequence over $[0, T_f]$ ($T_f > t$ where t is the current time, and T_f the time into

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$\eta(\cdot\,;\cdot)$	wave surface elevation at point $(\cdot)$ and time $(\cdot)$
ω	angular frequency of wave/oscillation
$\bar{a}_t(\omega)$, $\bar{a}_b(\omega)$	added mass variations for the top and bottom bodies, respectively, inclusive of infinite-frequency parts
$\varepsilon(x_A\,;t)$	measurement error in wave elevation at $(x_A\,;t)$
$\varepsilon(x_B\,;t+t_p)$	prediction error at $(x_B\,;t+t_p)$ as a result of measurement error at $(x_A\,;t)$
$\varepsilon_l(t)$, $\varepsilon_a(t)$	errors in resistive force $F_l(t)$ and reactive force component $F_a(t)$ due to measurement error in wave elevation at $(x_A\,;t)$
A	incident wave amplitude
$a_c(\omega)$, $b_c(\omega)$	added mass and radiation damping coefficients representing the frequency-dependent radiation coupling between the top and bottom bodies
$b_t(\omega)$, $b_b(\omega)$	radiation damping variations for the top and bottom bodies, respectively
c_{dt}, c_{db}	linearized, constant viscous damping coefficients for the top and bottom bodies, respectively
D	constant damping load applied on the relative heave oscillation
d	x-separation between the up-wave profile measurement location and device centroid
F_l	force applied by a power take-off actuator between the top and bottom bodies
F_{ft}, F_{fb}	exciting forces on the top and bottom bodies, respectively
$h_e(t)$	non-causal impulse response function (with respect to incident wave profile at body centroid x_B) representing the exciting force
h_l	impulse response function defining linear, unidirectional propagation of an impulsive wave elevation through a distance d
$h_l(t)$, $h_a(t)$	impulse response functions representing the resistive and reactive parts of the 'load' force applied by the power take-off
$h_o(t)$	impulse response function leading to hydrodynamically optimum relative heave velocity
$k(\omega)$	wave number; related to angular frequency ω through the dispersion relation
k_t, k_b	stiffness constants determining the restoring forces on the top and bottom bodies, respectively
$L(\omega)$, $N(\omega)$	resistive and reactive parts of the impedance generated by the power take-off
m_t, m_b	in-air masses of the top and bottom bodies, respectively
P_w	average power absorbed under approximate near-optimal control
P_{in}	time-averaged incident wave power per unit crest-length
$R_i(\omega)$, $C_i(\omega)$	equivalent hydrodynamic damping and reactance components 'acting on' the relative oscillation between the two bodies
v_r, x_r	relative heave velocity and displacement between the top and bottom bodies
v_t, v_b	heave oscillation velocities of the top and bottom bodies, respectively
v_{gmn}	minimum group velocity as determined by the high-frequency 'cut-off' for a realistic wave spectrum
v_{gmx}	maximum group velocity as determined by the low-frequency 'cut-off' for a realistic wave spectrum
x_A	x-coordinate along the propagation direction where wave profile measurement is made
x_B	x-coordinate along the propagation direction where the device centroid is located

the future up to which a latching sequence is to be determined). The formulation leads to a 2-point boundary value problem, and for this reason, the exciting force on the body needs to be predicted $(T_f - t)$ into the future. A declutching approach was developed later mainly for devices with longer resonance periods than the prevailing energy periods [15,16]. With either latching or declutching the sudden application or release of large forces/moments can lead to transient vibrations through the entire system, which, in addition to the force/moment load magnitudes, should be considered early in the design process.

Early reactive control approaches were later generalized to multiple-mode devices, and it was also shown that maximum power capture was possible under impedance matching conditions, wherein the load impedance matrix would be the complex-conjugate of the body impedance matrix in water [17–19]. This approach involves exchange of reactive power and energy storage, but enables greater energy absorption annually as long as the power take-off mechanism is capable of applying reactive loads. Non-real time applications of this approach include 'peak-frequency tuning' where the incoming wave spectra are sampled at regular intervals (e.g. 10 min) and the load impedance is adjusted so as to approach impedance matching at the spectral peak frequency. This strategy was recently tested on the wave star device [20] and a 2–3-fold improvement in annual power production was reported.

Still greater improvements are possible if optimal impedance matching can be achieved in real time on a wave-by-wave basis. However, as is well known, wave-by-wave implementation of such control is not possible without prior knowledge or estimation of the device velocity and force some duration into the future. The underlying cause of this difficulty is that the radiation force felt by an oscillating body (due to the waves it creates with its own oscillation) is causal. Thus, for the impulse response function h_r describing this force, for $t < 0$, $h_r(t) = 0$. Causality of h_r implies that the Fourier transform of h_r is an analytic function in the top half plane, so that the real and imaginary parts of its Fourier transform are constrained together by the Kramers–Kronig relations [21,22]. For impedance matching, the real part needs to be matched, and the imaginary part needs to be canceled. Because $h_r(t)$ is also real-valued, $h_r = h_a + h_b$, where h_a is an odd function, and h_b is an even function. Moreover, the two add together to zero for $t < 0$ and to h_r for $t \geq 0$. To implement impedance matching wave-by-wave, however, as mentioned, $-h_a$ and h_b need to be synthesized and utilized independently. Since both are non-causal, i.e. $h_a, h_b \neq 0$, $t < 0$, velocity information from the future is required in the generation of the control force at any instant t. This situation is discussed in [23,1]. In addition, wave action on the free surface is a continuum process over a distributed medium, and further, the body has a finite geometric size. Therefore, the body begins to experience the exciting force before the wave reaches it and before there is a wave elevation detected at its centroid. Thus, the impulse response function describing the exciting force with respect to the surface elevation at the body centroid is non-causal [1].

Hence, application of a control strategy seeking wave-by-wave impedance match requires the knowledge of the incident wave profile (and based on it, the device oscillation) in advance. In practice, future information over 25–30 s may be sufficient for a reasonable approximation to the required control force. Thus, approximate approaches based on time-series analysis were applied in [24,25]. More systematic forecasting techniques have developed in recent years [26]. A number of other control approaches seeking realistic solutions have also been investigated during the past decade [27–29]. A purely causal strategy based within a Linear Quadratic Gaussian formulation was discussed in [30]. A comparative evaluation of several control strategies was presented in [31]. Direct use of wave profile measurements some distance up-wave for evaluating the control force at the current time was considered in [32].