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Second-order Taylor expansion boundary element method for the second-order wave radiation problem

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ABSTRACT

A novel Boundary Element Method (BEM) named the second-order Taylor Expansion Boundary Element Method (the 2nd order TEBEM) is developed for the solution of the second-order wave radiation velocity potential and sum-frequency wave loads for floating bodies. The radiation condition is enforced by a hybrid method of the multi-transmitting formula and damping zone. For the interior domain problem of a cube and a sphere, numerical results demonstrate that the 2nd order TEBEM can accurately solve the first and second-order gradients of velocity potential on the no-smoothed and smoothed boundary compared to the low-order BEM. The double frequency forces acting on the truncated cylinder are calculated under finite water depth. The agreement between the 2nd order TEBEM and others' numerical results is good. Moreover, all of the singular integrals in the 2nd order TEBEM can be solved analytically, so its implementation is much easier compared to the high-order BEM.

1. Introduction

Although many researchers pay attention to fully nonlinear numerical model, a facile and accurate numerical scheme for weakly nonlinear wave problem is needed especially for complex structures. Second-order wave forces cause the mean, difference frequency and sum-frequency effects, such as the added resistance of ships, slow drift motion of offshore floating structures, vertical sum-frequency loads on the Tension Leg Platforms (TLP) and so on. The second-order wave diffraction and radiation wave-body interaction problems have been discussed by both analytical and numerical method. Faltinsen and Loken [1], Molin [2], and Lighthill [3] calculate the second-order wave force through the fictitious radiation potential without solving the second-order potential. Nevertheless, this method can only give out of the total forces and cannot be used to calculate the local quantities such as the hydrodynamic pressure and wave elevation due to the second-order velocity potential.

The Finite Element Method (FEM) is usually used to solve the Boundary Value Problem (BVP). Compared to the Boundary Element Method (BEM), the coefficient matrix of the FEM is banded and has a smaller memory requirement. Ma et al. [4], Hu et al. [5] and Wang and Wu [6,7] obtain a variety of fully nonlinear and weakly nonlinear problems solution by this method. However, a major challenge for the FEM is the mesh generation. For a complex structure, a complicated mesh generator is commonly required to follow the motion of the structure and the wave. Hence, BEM is applied in the present paper.

The BVP can be solved in both frequency and time domain, numerical analysis in frequency domain can be found in Loken [8], Eatock Taylor and Hung [9] and Kim and Yue [10], which compute the second-order wave loads of the single cylinder. Maniar and Newman [11], Evans and Porter [12], Malenica et al. [13] and Kashiwagi and Ohwatari [14] investigate the interaction among the cylinders. Method in frequency domain is generally used to solve the linear problems, while a direct solution of the hydrodynamic problem in the time domain is more convenient for high-order problems when needing to consider the transient effects.

In time domain, the scheme to solve the BVP by the BEM can be divided into two kinds. The first one is to use the Green function which satisfies the free surface condition. However, the Boundary Integral Equation (BIE) involves the memory term and encompasses all prior information. The memory effect would become too large for long time simulation especially. The other scheme is to utilize the Rankine source. This Green function does not satisfy the free surface condition, so it is necessary to distribute the source on the free surface. Compared to the first one, the Rankine source method removes the explicit memory effect from the integral equation. Typical works of this method are finished by Isaacson and Cheung [15,16] calculating the second-order wave loads on a single cylinder. Nevertheless, the solution requires truncating the fluid field at some finite distance. A proper numerical Damping Zone

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(DZ) is used to absorb the wave and prevent the reflection phenomenon on the truncated surface represented in Orlanski [17], Boo [18] and Clamond et al. [19]. But the efficiency of the DZ is dependent on the ratio between the length of the beach and wave length. So more panels would be needed on the free surface if wave length is large. Xu and Duan [20], Zhang and Duan [21] propose the Multi-Transmitting Formula (MTF) coupled with the DZ to simulate the outgoing waves on the control surface, which can reduce the length of the beach effectively.

When a weakly nonlinear wave body problem is studied, perturbation method based on the Taylor expansion of the body surface and free surface about the mean position of stationary body and the calm water surface is applied. One difficulty in numerical calculation is to solve the high-order derivatives in the body and free surface conditions. Teng [22] and Bai [23] use the Stokes theorem to eliminate the second-order derivatives and substitute by the first-order ones, which have to calculate the integration along its waterline and the Green function involving its first-order derivatives on body surface. Shao and Faltinsen [24] calculates the second-order wave loads under the body-fixed coordinated system without the derivatives of the velocity potential on the body boundary condition. However, the second-order derivatives still exist on the other surfaces. Many numerical methods also can be used such as the high-order Finite Difference Method (FDM), high-order BEM and the B-spline based on BEM. These methods obtain the first and second-order derivates involving the values of the potential at the corners and converge slowly. In addition, the desingularized panel method can remove the difficulty associated with the singular behavior of the Green function, so we can compute the derivatives by differentiating the Green function. While this mean only can be used in wave body interactions without sharp corners [25].

Hence, this paper proposes a novel BEM, named the 2nd order Taylor Expansion Boundary Element Method (the 2nd order TEBEM), which can directly solve the velocity potential, the first and second-order derivatives at the centroid of the discretization panels. This method is based on the framework of the low-order direct BEM to solve the BIE, which mainly applies the Taylor expansion to the dipole and source strength of the BIE, reserves the second and first-order derivatives respectively, and finally utilizes the corresponding tangential derivatives respecting to the field points on the boundary elements to form the closed equations. This paper shows the application of the TEBEM for solving second-order radiation problems. Mathematical formulation of radiation problem is shortly reviewed in Section 2. Theory and numerical procedure for the TEBEM are discussed in Section 3. Numerical treatment of radiation condition and free surface condition are shown in Section 4. Coordinate transformation between the earth and local coordinate system is shown in Section 5. In Section 6, the numerical issues of cylinder radiation problem under finite water depth will show the superiority of the 2nd order TEBEM.

2. Mathematics formulations for the wave radiation problem

The coordinate system and the computation domain are illustrated in Fig. 1. The boundary surface S are composed by the mean position of the body S_H, the mean free surface S_F, the horizontal bottom surface S_B and the artificial boundary surface S_C, which divides the fluid domain into the inner and outer regions. When the water is assumed incompressible and inviscid, and the flow is irrotational, the motion of the water can be described by the velocity potential φ satisfying the Laplace equation.

$$\nabla^2\varphi = 0 \tag{1}$$

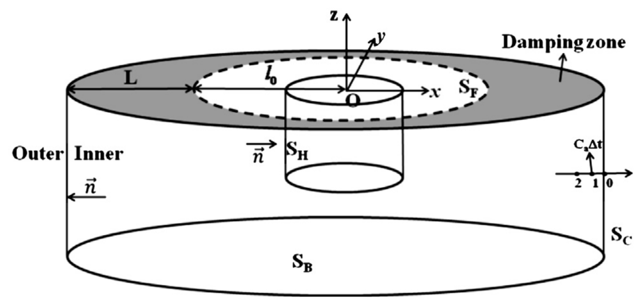

Fig. 1. The sketch of coordinate system and computation domain.

Since the pressure on the transient free surface $z = \eta(x, y, t)$ is always the atmosphere pressure, the substantial derivative of the pressure is equal to zero. Mathematically it can be expressed as:

$$\frac{\partial^2\varphi}{\partial t^2} + g\frac{\partial\varphi}{\partial z} + 2\nabla\varphi\nabla\frac{\partial\varphi}{\partial t} + \frac{1}{2}\nabla\varphi\cdot\nabla(\nabla\varphi\cdot\nabla\varphi) = 0 \tag{2}$$

where g is the acceleration of gravity.

The body boundary condition on the instantaneous wetted body surface

$$\frac{\partial\varphi}{\partial N} = \vec{V}\cdot\vec{N} \tag{3}$$

to make sure that fluid particles cannot penetrate the body surface. $\vec{N}$ is the normal vector on the transient body surface. $\vec{V}$ represents the velocity of the rigid body motion under the earth coordinate system.

The sea bottom is assumed to be horizontal rigid plane and the boundary condition is

$$\frac{\partial\varphi}{\partial n} = 0 \tag{4}$$

A proper radiation condition on the artificial boundary surface should be imposed to ensure that radiation waves cannot be reflected into the calculation domain over the finite time. The detailed discussion will be presented in Section 4.

Under the assumption of weakly nonlinear wave and the perturbation method, Eq. (2) can be satisfied on the calm water surface through the Taylor expansion. Correspondingly, the potential and body motion can be written as

$$\varphi = \varepsilon\varphi^{(1)} + \varepsilon^2\varphi^{(2)} + \cdots \tag{5}$$

$$\vec{\xi} = \varepsilon\vec{\xi}^{(1)} + \varepsilon^2\vec{\xi}^{(2)} + \cdots \tag{6}$$

$$\vec{\alpha} = \varepsilon\vec{\alpha}^{(1)} + \varepsilon^2\vec{\alpha}^{(2)} + \cdots \tag{7}$$

where ε is a perturbation parameter which is usually related to wave slope. The superscripts (1) and (2) denote the first and second-order potential and body motion. $\vec{\xi} = (\xi_1, \xi_2, \xi_3)$ is the translational displacement vector and $\vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ is the angular displacement vector.

So the first and second-order potentials satisfy the following boundary condition within the fluid domain:

$$\frac{\partial^2\varphi^{(j)}}{\partial t^2} + g\frac{\partial\varphi^{(j)}}{\partial z} = f^j\,(j = 1, 2;\quad \text{on}\, S_F) \tag{8}$$

$$\frac{\partial\varphi^{(j)}}{\partial n} = f_j'\,(j = 1, 2;\quad \text{on}\quad S_H) \tag{9}$$

$$\frac{\partial\varphi^{(j)}}{\partial n} = 0\,(j = 1, 2;\quad \text{on}\quad S_B) \tag{10}$$

where $f^1 = 0,\quad f^2 = -2\nabla\varphi^{(1)}\cdot\nabla\left(\frac{\partial\varphi^{(1)}}{\partial t}\right) + \frac{1}{g}\frac{\partial\varphi^{(1)}}{\partial t}\cdot\frac{\partial}{\partial z}\left(\frac{\partial^2\varphi^{(1)}}{\partial t^2} + g\frac{\partial\varphi^{(1)}}{\partial z}\right).$