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Domain reduction for Benders decomposition based global optimization

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ABSTRACT

While domain reduction has been successfully applied in branch-and-bound based global optimization over the last two decades, it has not been systematically studied for decomposition based global optimization, which is usually more efficient for problems with decomposable structures. This paper discusses integration of domain reduction in Benders decomposition based global optimization, specifically, generalized Benders decomposition (GBD) and nonconvex generalized Benders decomposition (NGBD). Revised GBD and NGBD frameworks are proposed to incorporate bound contraction operations or/and range reduction calculations, which can reduce the variable bounds and therefore improve the convergence rate and expedite the solution of nonconvex subproblems. Novel customized bound contraction problems are proposed for GBD and NGBD, and they are easier to solve than the classical bound contraction problems because they are defined on reduced variable spaces. The benefits of the proposed methods are demonstrated through a gas production operation problem and a power distribution system design problem.

1. Introduction

Global optimization for nonlinear programming (NLP) and mixed-integer nonlinear programming (MINLP) has been applied widely to various engineering problems, ranging from product distribution, infrastructure design to process design and control (e.g., Floudas, 1995; Grossmann, 2002; Biegler and Grossmann, 2004). The models of engineering problems, represented by both linear and nonlinear constraints with continuous or/and discrete decisions, usually lead to nonconvex mathematical programming problems, for which local optimization methods can fail to find and verify global optimal solutions.

It is well-known that, classical branch-and-bound based global optimization methods suffer from curse of dimensionality (i.e., the solution time scales exponentially with the problem size in the worst case). Therefore, they are impractical for large-scale problems that may result from optimization of complex engineering systems or/and explicit consideration of uncertainties. A natural idea to deal with a large-scale problem is to break the problem into smaller subproblems, which are convex or nonconvex but can be solved efficiently with branch-and-bound based global optimization. There are two types of decomposition strategies for solving large scale nonconvex problems. One is Lagrangian decomposition based global optimization (Fisher, 1981; Dür and Horst, 1997; Caroe and Schultz, 1999; Karuppiah and Grossmann, 2008), which is incorporated in a branch-and-bound framework where only a subspace of the problem is partitioned. The other is Benders decomposition based global optimization (Benders, 1962; Geoffrion, 1972; Li et al., 2011b), which does not require an explicit branch-and-bound procedure but requires certain problem structures. In Benders decomposition based global optimization, one or multiple nonconvex subproblems need to be solved at each iteration, so the efficiency of optimization relies on how fast each nonconvex subproblem can be solved and how fast the convergence to a global solution can be reached. Benders decomposition based optimization is well known to be advantageous for scenario based stochastic programming (Birge and Louveaux, 2011), and it has also been applied to a wide range of deterministic optimization problems, such as process synthesis (Acevedo and Pistikopoulos, 1998), process design and operation (Li et al., 2011a; Li and Li, 2016; Frank and Rebennack, 2015), circle cutting (Rebennack, 2016), etc.

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Domain reduction techniques can benefit branch-and-bound based global optimization, because they can reduce the search domain, which result in tighter convex relaxations and reduced number of nodes that are to be explored in the branch-and-bound search. Tawarmalani and Sahinidis (2004) developed a theoretical global optimization framework based on Lagrangian duality for branch-and-bound search, and it can be customized to yield a variety of domain reduction techniques in the literature. These domain reduction techniques can be classified into two types. One type extensively utilizes the primal and dual solutions of convex relaxation subproblems to reduce the variable ranges, and it does not require solving extra domain reduction subproblems (unless some sort of probing procedure is needed) (Thakur, 1991; Ryoo and Sahinidis, 1996). The other type requires solving extra convex optimization subproblems to reduce the variable ranges (Maranas and Floudas, 1997; Zamora and Grossmann, 1999; Castro and Grossmann, 2014). In order to distinguish the two types of techniques, we call the former *range reduction calculation* and the latter *bound contraction operation*. While bound contraction operation can usually reduce variable ranges more effectively, it requires much more computing time and therefore cannot be performed as frequently as range reduction calculation do.

In the light of successful integration of domain reduction in branch-and-bound based global optimization, this paper discusses the integration of domain reduction in generalized Bender decomposition (GBD, Geoffrion, 1972) and nonconvex generalized Benders decomposition (NGBD, Li et al., 2011b). At the best of our knowledge, this is the first attempt in the literature to integrate domain reduction in Benders decomposition based global optimization methods. Customized bound contraction operations and range reduction calculations are proposed in the paper to accelerate the solution of nonconvex subproblems or/and reduce the number of nonconvex subproblems to be solved, for GBD and NGBD.

Nonconvex NLPs/MINLPs that can be solved by GBD or NGBD are expressed in the following separable form:

$$\begin{aligned} \min_{z,z_0} \quad & f_1(z)+f_2(z_0) \\ \text{s.t.} \quad & g_1(z)+g_2(z_0) \le 0, \\ & z \in Z, \quad z_0 \in Z_0, \end{aligned} \tag{P0}$$

where f_1 and f_2 are scalar-valued functions, g_1 and g_2 are vector-valued functions. Note that (P0) can always be rewritten into the following form:

$$\begin{aligned} \min_{x,y_0} \quad & c^{\mathrm{T}}x \\ \text{s.t.} \quad & Ax+By_0 \le d, \\ & x \in X, \quad y_0 \in Y, \end{aligned} \tag{P1}$$

where set $X = \{x \in \Omega \subset \mathbb{R}^{n_x} | \psi(x) \le 0\}$ are defined with vector-valued function $\psi : \Omega \to \mathbb{R}^{m_\psi}$ that only contains nonlinear constraints, set $Y = \{y_0 \in \Phi \subset \mathbb{R}^{n_{y_0}} | \varphi(y_0) \le 0\}$ are defined with vector-valued function $\varphi : \Phi \to \mathbb{R}^{m_\varphi}$. We call variables y_0 linking variables, given that (P1) can be separated into a number of relatively easy subproblems if these variables are fixed. Constraint $Ax+By_0 \le d$ contains all linear constraints in the problem, including those link x and y_0 and those contain only x. The detailed procedure for transforming (P0) to (P1) is provided in Appendix A.

Obviously, Problem (P1) can be further reformulated into the following form, by introducing extra variables y:

$$\begin{aligned} \min_{x,y,y_0} \quad & c^{\mathrm{T}}x \\ \text{s.t.} \quad & y-y_0 = 0, \\ & Ax+By \le d, \\ & x \in X, \quad y_0 \in Y. \end{aligned} \tag{P}$$

In this reformulation, the constraints that link y_0 and other variables become linear equality constraints that are free from any problem parameters. We call these constraints linking constraints in the paper. Due to the linking constraints, y is a duplicate of y_0, so its value has to be within Y. Later we will show that reformulation to (P) is necessary for efficient domain reduction within Benders decomposition based optimization framework. Therefore, formulation (P) will be considered for the rest of the paper.

Remark 1. (x^*, y_0^*) is an optimal solution of Problem (P0) if and only if (x^*, y_0^*) is an optimal solution of Problem (P1). (x^*, y_0^*) is an optimal solution of Problem (P1) if and only if (x^*, y^*, y_0^*) (where $y^* = y_0^*$) is an optimal solution of Problem (P).

For convenience of subsequent discussion, we make the following assumption for sets X and Y:

Assumption 1. Sets X and Y are nonempty and compact.

The remaining part of this paper is organized as follows. Section 2 introduces the classical GBD method and discusses why reduced variable domain can benefit GBD, and then two bound contraction operations are developed and incorporated in GBD for improved solution efficiency. Section 3 introduces the standard NGBD method in the context of a multi-scenario version of problem formulation, and develops customized bound contraction operations and range reduction calculations for NGBD. Section 4 demonstrates the benefits of the proposed domain reduction methods for GBD and NGBD, through two optimization problems for energy systems. The paper ends with conclusions in Section 5.