Applied Ocean Research 51 (2015) 255–267

Contents lists available at ScienceDirect

Applied Ocean Research

journal homepage: www.elsevier.com/locate/apor

A wavelet-based test for swell stationarity

Kevin C. Ewans

Shell International Exploration and Production, The Netherlands

ARTICLE INFO

Article history:
Available online 11 February 2015

Keywords:
Swell stationarity
Wavelet transform
LNG barge
Offloading

ABSTRACT

The temporal behaviour of swell is examined using the wavelet transform, and a test of stationarity using the wavelet transform is described. The method is demonstrated on time series data from a Directional Waverider buoy deployed at Duck, North Carolina, and it is shown that non-stationary sea states are usually associated with local wind-sea growth rather than significant changes in the swell component, which can be considered stationary for at least as long as the 160-min records. This is close to the 3-h duration that is typically assumed for a stationary sea state. The stationary character of swell is an important result for offloading from an LNG barge to carrier.

1. Introduction

Most offshore operations can only be undertaken when the sea state is within prescribed limiting conditions. For example, offloading operations, such as ship to ship transfers, require relatively quiescent metocean conditions; but for many locations, quiescent conditions are nevertheless associated with a more or less constant level of background swell, which can still excite significant vessel motions. In the case of large vessels involved in offloading, such as from an LNG barge to carrier, the characteristics of the swell, particularly its temporal variability, can be more significant than those of the total sea state. It is therefore of interest to understand the spectral characteristics of the low-frequency band or swell component of the wave spectrum as well its temporal behaviour, particularly over durations of the order of the offloading operations, some 12–24 h, over which time the sea state characteristics, such as the significant wave height, may change. Lucas and Guedes Soares [29] focus on the description of the swell spectrum, while in this paper the focus is on the temporal variability of these spectral characteristics over durations of the order of a sea state.

Sea states are generally assumed to be stationary. The strict mathematical definition of stationarity is that the joint probability distribution does not change in time or space, but this definition is relaxed somewhat in the weak or wide sense definition of stationarity, which requires only that the mean and variance to not vary with time. Geophysical processes often have or can be assumed to have zero mean values. For example, the sea surface elevation as measured by a Waverider buoy, which does not respond to tidal periods, will effectively have a zero-mean. Thus, it is sufficient to test only that the variance is stationary in examining for weak sense stationarity. The strictly stationary definition would require that the statistical character of the sea state does not change over the duration of the sea state, which is often assumed to be three hours, and over some spatial domain. Indeed, most of the statistical parameters that are estimated and used to quantify a sea state are implicitly based on the assumption that the sea state is stationary. Such quantities as the wave spectrum, and parameters derived from it, such as the significant wave height and mean wave periods, are associated with the assumption that the sea state is stationary. The spectra and parameters are often estimated from sample records of 20–30 min in duration that have been recorded at three hour time steps, or they may be available from numerical model predictions that have three or even six hour time steps. As a consequence, the stationarity assumption is often extended to also apply to the duration of the time step.

There is little to be found in the literature on studies that have investigated the stationarity of sea states. Of note, Haver and Moan [1] found in an examination of 384 20-min sea states, that there was an increased probability that the sea state would be non-stationary when the significant wave height was larger than four metres. This seemed to have some support from Doucette et al. [2], who found the expected duration of a stationary sea state to decrease exponentially with increasing significant wave height. Doucette et al. [2] examined the stationarity of significant wave height data from wave radar measurements made on the Frigg platform located between the Shetland Islands and Norway. The significant wave height values were calculated at 20 min intervals from wave data recorded at 2 Hz sampling frequency, and the durations for which the significant wave height values were deemed to be stationary was established by performing a change-point analysis. The durations were found to be exponentially distributed and it was concluded that three hours could be considered the mean duration of a sea state for the location. Labeyrie [3] also referred to the Frigg data and presented almost the same results as Doucette et al. [2],

E-mail address: Kevin.Ewans@shell.com

0141-1187/$ – see front matter © 2015 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.apor.2015.01.005

except Labeyrie concluded that the expected duration was independent of the level of the sea state. Tournadre [4] also concluded that the energy of the sea state and the duration over which it could be considered stationary were independent. All of these studies were based on parameters representing the complete sea state; there does not appear to have been any systematic studies of the stationarity of the swell component within sea states. It is reasonable to expect that sea states that are dominated by swell or the swell component itself might be stationary for longer periods than three hours.

While it is clear that the duration over which a sea state can be considered is variable and that a sea state might not be stationary for a given duration, even within a stationary sea state there is significant variability. A large number of studies have focussed on the variability within the sea state, addressing issues such as rogue waves, wave grouping, and wave breaking, and employing a variety of techniques involving time–frequency analyses, such as the short-time Fourier transform [5] and higher-order versions [6,7], the Stockwell transform [8], the Hilbert–Huang transform [9,10], and the wavelet transform (e.g. [11]).

In this paper we employ the wavelet transform to investigate the temporal behaviour of swell. Many studies investigating temporal variability with the wavelet transform have been reported. Massel [11] provides a good overview of the fundamentals of the wavelet transforms and their application to ocean waves, providing examples of Morlet transforms of time series resulting from recording the wind induced deep water waves, waves breaking on the tropical coral reefs and laboratory waves. The Morlet transform has been used extensively for ocean wave analyses. Liu [12] used the Morlet wavelet transform to analyse wave data collected during the 1990 SWADE (Surface Wave Dynamics Experiment) program, and documented new insights on wave grouping parameterizations, phase relations during wind wave growth, and wave breaking characteristics. Huang [13] discusses wave parameters and functions calculated from the Morlet wavelet transform using wave data measured at an offshore coastal observation. Shen et al. [14] used the Morlet wavelet transform to show the intermittent nature of high frequency waves in the equilibrium range of the energy spectrum. Ma et al. [15] developed a new method for separation of 2D incident and reflected waves using the Morlet wavelet transform and demonstrated the efficiency and accuracy of this method are demonstrated using numerical simulated data. Donelan et al. [16] and Krogstad et al. [17] used the Morlet wavelet transform to investigate the directional variability within a sea state.

There are also a number of papers on the application of the Morlet wavelet transform in rogue wave studies. Liu and Mori [18] presented Morlet spectra for wave records with rogue waves recorded in the Sea of Japan, and Schlurmann [19] compared Morlet spectra against spectra from the short-time Fourier transform and Hilbert–Huang transform for rogue wave records made in the Sea of Japan. Krogstad et al. [20] used the wavelet-based directional method to show that rogue waves tend to have directions close to the mean wave direction and to have a low directional spread. Ewans and Buchner [21] and Christou et al. [22] used the Morlet transform to investigate spectral properties of a large wave measured in the laboratory, noting the occurrence of increased spectral levels over a broad range of frequencies at the time of the large wave.

This paper reports an investigation of the temporal behaviour of swell, primarily focussed on stationarity aspects. Von Sachs and Neumann [23] developed a test for stationarity of time series data based on the Haar wavelet, but in this paper the Morlet wavelet transform is used, due to its more intuitive frequency interpretation. The strength of the Morlet wavelet spectrum for examining the temporal behaviour of swell is demonstrated, and a wavelet-based technique to test for stationarity is developed. The method is applied to time series data recorded with a Directional Waverider buoy moored at the US Corp of Army Engineers' Field Research Facility (FRF) at Duck, North Carolina. The stationarity of wave records and specifically the swell component is evaluated, and records that are deemed to be non-stationary are examined to ascertain the reason for their non-stationarity.

Section 2 describes the use of the wavelet transform for investigating the temporal variability of sea states and swell, and some examples are given for the FRF data. The use of the wavelet transform for examining sea states and swell stationarity is given in Section 3. Section 3 also includes the results of the application of the wavelet stationarity test to the FRF data. Conclusions are drawn in Section 4.

2. The wavelet transform for examining temporal characteristics

2.1. Background

The difficulty in any investigation of swell within a given sea state is that the sea state is generally complicated by the presence of higher frequency energy associated with the local wind-sea. In order to enable examination of the swell component, a method to distinguish it from the higher frequency components is needed; this is generally achieved by partitioning the wave spectrum (for example [24,25]), but a wavelet analysis is better suited for temporal evaluation. In particular, the Morlet transform is well suited for this purpose, due to its resemblance to the conventional Fourier spectrum and therefore the ease with which it can be interpreted (e.g. [11]). Application of the Morlet transform to time domain wave records allows not only the swell component to be easily identified but also its variation with time.

Wavelet analysis involves a convolution of a real time series with a set of functions that are derived from a "mother wavelet", ψ_0, with the properties:

$$\int_{-\infty}^{\infty} \psi_0(t)dt = 0$$

$$\int_{-\infty}^{\infty} |\psi_0(t)|^2 dt = 1$$

The Morlet wavelet is defined by:

$$\psi(t) = \frac{1}{\sqrt{\pi f_b}} \exp(2\pi i f_c t) \exp\left(-\frac{t^2}{f_b}\right)$$

where, f_b is a window width parameter, and f_c is the centre frequency.

By comparison with a Gaussian function it can be recognised that $f_b = 2\sigma^2$, where σ^2 is the variance of the Gaussian function. Further, it is clear that f_c is the frequency under consideration. A plot of the real part of a Morlet wavelet is given in Fig. 1.

The continuous Wavelet transform is defined by

$$W(s,p) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} \psi^* \left(\frac{(t-p)}{s}\right) f(t)dt$$

where s is the scale, and p is the shift.

The wavelet spectrum is then given by $|W(s,p)|^2$ with appropriate scaling. The continuous Morlet wavelet transform of the signal is the scaled Fourier transform convolved with the Gaussian window, which would be equivalent to conventional Fourier analysis applied to the signal weighted by a Gaussian window; the Gaussian function in the Morlet wavelet is effectively the window function in traditional Fourier analysis. A distinguishing feature of the continuous Morlet wavelet transform is that the size of the Gaussian