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Analysis of loads, motions and cavity dynamics during freefall wedges vertically entering the water surface

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ABSTRACT

In this paper, theoretical models are developed and numerical methods are used to analyze the loads, motions and cavity dynamics for freefall wedges with different deadrise angles vertically entering the water surface at Froude numbers: $1 \leq Fn < 9$. The time evolutions of the penetration depth, the velocity and the acceleration are analyzed and expressed explicitly. The maximum and average accelerations are predicted. The theoretical results are compared with numerical data obtained through a single-fluid BEM model with globally satisfactory agreement. The evolution of the pressures on the impact side is investigated. Before flow separation, gravity and the acceleration of the wedge have negligible influence on the pressure on the impact side for large Froude numbers or small deadrise angles; with increasing the deadrise angle or decreasing Froude number, the effects of gravity and the acceleration of the wedge tend to become more important. Global loads, with the main emphasis on the drag coefficient, are also studied. It is found that for the light wedge, the transient drag coefficient has slow variation in the first half of the collapse stage and rapid variation in the last half of the collapse stage. For the heavy wedge, the transient drag coefficients vary slowly during the whole collapse stage and can be treated as constant. The characteristics of the transient cavity during its formation are investigated. The non-dimensional pinch-off time, pinch-off depth and submergence depth at pinch-off scale roughly linearly as the Froude number.

1. Introduction

When a freefall wedge vertically enters the water surface, it may experience slamming, transition, collapse, and post-closure stages [1]. The slamming stage is the initial stage of the wedge penetrating the water surface. During this stage, the water rises up, jets are formed at the body sides and impulse loads may occur. By impulse loads we mean that the local pressures on the impact side (the side contacting the water) of the wedge and the global vertical load acting on the wedge is impulsive in time. The pressure distribution on the impact side is also spatially 'impulsive' with a localised region of high pressure. The magnitude of the impulse loads and the pressure distribution on the impact side are also of interest. By assuming a constant entry speed and neglecting the effect of the gravity, Dobrovol'skaya [2] presented similarity solutions for the water entry of a wedge with any deadrise angle β. The similarity solution is not available in explicit form and numerical results were only presented for $\beta \geq 30^\circ$. Zhao and Faltinsen [3] developed advanced numerical methods and presented numerical results for deadrise angles from 4° to 81°. Zhao and Faltinsen concluded: when $\beta \geq 45^\circ$, the maximum pressure is at the apex of the wedge; when β is less than 30° approximately, high impulse pressures are concentrated on a small area near the root of the jet. Carcaterra et al. [4,5] developed analytical models for the hydrodynamic force and wedge motion during the slamming stage, which assume a constant entry speed and neglect gravity force. During the water entry of freefall wedges, both gravity force and wedge acceleration influence the wedge motion and/or the hydrodynamic force. For a low-speed water entry, the gravity force is comparable to or even larger than the hydrodynamic force and therefore has an important influence on the wedge motion; for a high-speed water entry, the freefall wedge experiences large vertical accelerations and therefore the added mass force becomes important. In the present paper, the effects of gravity and accelerations are taken into account. Special attention is given on how gravity and the vertical acceleration influence the pressures on the impact side of the wedge. When the jet reaches the knuckle of the wedge, non-viscous flow separation occurs. The jet breaks up into spray. When the root of the jet/spray leaves the knuckle of the wedge, a strong drop of the slamming force occurs and the transition stage starts. It is interesting how the pressures on the impact evolve during the transition stage. When the root of the spray is far away from the knuckle, an open cavity

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is formed on the top of the wedge which then collapses. The collapse stage ends at closure (pinch-off) of the cavity during which the hydrodynamic loads change slowly [1]. The evolution of the global hydrodynamic loads, the wedge motions and the evolution of the cavity are of particular interest. Wang and Faltinsen [6] investigated the evolution of the cavity numerically showing that: the cavity closure period is independent of the initial entry speed of the wedge; the submergence depth of the wedge at pinch-off increases approximately linearly with respect to the initial entry speed; and the cavity size is highly dependent on the mass with a larger mass causing a larger cavity. Here, we will discuss the mechanics behind these numerical findings.

In this paper, theoretical models are developed to describe the wedge motions and cavity dynamics of a freefall wedge vertically entering the water surface. The present study is focused on the kinematics and dynamics of the freefall wedge until the closure of the cavity. No consideration is given to the post-closure stage where air compressibility matters [1]. The time evolutions of the penetration depth, the velocity and the acceleration are analyzed and expressed explicitly. The maximum and average accelerations of the wedge are predicted. The theoretical analyses show the existence of a critical Froude number: for Froude number less than the critical Froude number the velocity of the wedge increases after the slamming stage; for Froude number larger than the critical Froude number the velocity decreases after the slamming stage. This is accordance with Wang et al.'s experiments [1]. An approximate cavity evolution model is proposed. The non-dimensional pinch-off time of the cavity scales linearly as the Froude number. Further, the characteristics of the transient cavity are predicted and it is found that the non-dimensional pinch-off depth and submergence depth (the distance from the still water level to the top side of the wedge) at pinch-off scale roughly linearly as the Froude number. The evolution of the pressures on the impact side and the drag coefficients are also extensively studied. It is found that for the light wedge the transient drag coefficients (see Eq. (21)) have slow variation in the first half of the collapse stage and rapid variation in the last half of the collapse stage; for the heavy wedge the transient drag coefficients vary slowly during the whole stage and can be treated as constant. Note that the numerical results presented in this paper are obtained by using the single-fluid BEM described in Ref. [1].

2. Loads and motions

A two-dimensional wedge with mass M, deadrise angle β, beam $2c_0$ vertically impacts the still water surface with initial entry speed V_0 as shown in Fig. 1. The surface tension σ can be neglected provided that the Weber number $W = \rho V_0^2 c_0/\sigma \gg 1$. Viscous effects may be excluded within the short duration of the impact and for high Reynolds number $Re = \rho V_0 c_0/\mu$. Further, we neglect the influence of the air flow. This assumption is in accordance with the study of Wang et al. [1]: for closure of 2D cavity, the air flow starts to play an important role just before the closure but its influence is very limited. Then the impact is characterized by the Froude number $F_n = V_0/\sqrt{gc_0}$, the mass ratio, and the deadrise angle β.

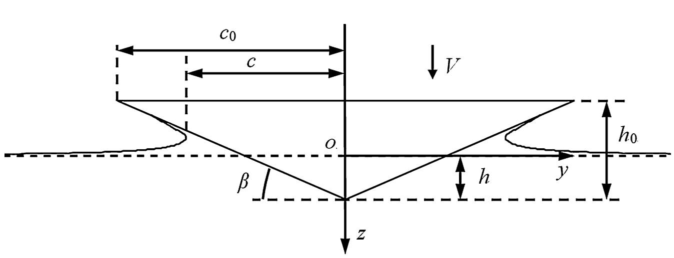

Fig. 1. Coordinate system and symbol definitions for the slamming stage.

Here, the mass ratio could be defined as the hydrostatic mass ratio $D_1 = \rho c_0 h_0/M$ or the hydrodynamic mass ratio $D_2 = 0.5\rho\pi c_0^2/M$. The hydrostatic mass ratio is the ratio between the mass of water displaced by the fully submerged wedge and the mass of the wedge, and the hydrodynamic mass ratio is the ratio between the added mass of the wedge (the high frequency added mass of a flat plate approximation is $0.5\rho\pi c_0^2$) and the mass of the wedge. The latter is more suitable to represent the mass ratio since the water impact is a dynamic process.

2.1. Global load and wedge motion

2.1.1. Slamming stage and transition stage

During the slamming stage, the water rises up due to the impact of the wedge, and jets are formed at the body sides as shown in Fig. 1. The penetration depth of the wedge satisfies

$$\frac{dh}{dt} = V. \tag{1}$$

Based on the conservation of fluid momentum, the slamming force (see page 299 in Ref. [7]) can be expressed as

$$-\frac{d}{dt}(A_{33}V).$$

Here A_{33} is the vertical added mass of the wedge, which can be represented as $A_{33} = 0.5\rho\pi c^2$. c is regarded as the equivalent half-wetted breadth and can be expressed as a water rise-up ratio λ times the measurement on the still water surface, i.e.

$$c = \lambda\left(\frac{h}{\tan\beta}\right). \tag{2}$$

Note that $\lambda = \pi/2$ corresponds to Wagner's approach [8] and $\lambda = 1$ von Kármán's approach [9]. If the Wagner–Sydow approximation [10] is used, the water rise-up ratio may be written as

$$\lambda = \left(\frac{\pi}{2\beta} - 1\right)\tan\beta. \tag{3}$$

Further, by neglecting the buoyancy and following Newton's second law the equation of motion of the wedge can be expressed as

$$M\frac{dV}{dt} = -\frac{d}{dt}(A_{33}V) + Mg. \tag{4}$$

We define the following non-dimensional variables: $\tilde{t} = t/\tau_0$, $\tilde{h} = h/c_0$, $\tilde{c} = c/c_0$, and $\tilde{V} = V/V_0$. Here $\tau_0 = c_0/V_0$. The equations of motion of the wedge are written in non-dimensional form as

$$\frac{d\tilde{h}}{d\tilde{t}} = \tilde{V} \tag{5}$$

and

$$\frac{d\tilde{V}}{d\tilde{t}} = -D_2\frac{d}{d\tilde{t}}(\tilde{c}^2\tilde{V}) + \frac{1}{Fn^2}. \tag{6}$$

Integrating (6) from 0 to $\tilde{t}$ yields

$$\tilde{V} = \frac{1}{1+D_2\tilde{c}^2}\left(1 + \frac{1}{Fn^2}\tilde{t}\right). \tag{7}$$

From (6), we can get

$$\frac{d\tilde{V}}{d\tilde{t}} = -\frac{2D_2\tilde{c}\lambda\tan^{-1}(\beta)\tilde{V}^2}{1+D_2\tilde{c}^2} + \frac{1}{1+D_2\tilde{c}^2}\frac{1}{Fn^2}. \tag{8}$$

Substituting (7) into (5) and integrating from 0 to $\tilde{t}$, the following relation between $\tilde{c}$ and $\tilde{t}$ is obtained

$$\left(\tilde{c} + \frac{1}{3}D_2\tilde{c}^3\right)\lambda^{-1}\tan\ \beta = \tilde{t} + \frac{\tilde{t}^2}{2Fn^2}. \tag{9}$$

When the root of the jet reaches the knuckle of the wedge, the slamming stage ends and the transition stage starts. This can