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Stability and liquefaction analysis of porous seabed subjected to cnoidal wave

Xiang-Lian Zhou[a,b,*], Jun Zhang[a,b], Jian-Hua Wang[b], Yun-Feng Xu[b], Dong-Sheng Jeng[b,c]

[a] State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, Shanghai 200240, China
[b] Centre for Marine Geotechnical Engineering, Shanghai Jiao Tong University, Shanghai 200240, China
[c] Griffith School of Engineering, Griffith University Gold Coast Campus, Queensland, QLD 4222, Australia

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ABSTRACT

Cnoidal wave theory is appropriate to periodic wave progressing in water whose depth is less than 1/10 wavelength. However, the cnoidal wave theory has not been widely applied in practical engineering because the formula for wave profile involves Jacobian elliptic function. In this paper, a cnoidal wave-seabed system is modeled and discussed in detail. The seabed is treated as porous medium and characterized by Biot's partly dynamic equations (u–p model). A simple and useful calculating technique for Jacobian elliptic function is presented. Upon specification of water depth, wave height and wave period, Taylor's expression and precise integration method are used to estimate Jacobian elliptic function and cnoidal wave pressure. Based on the numerical results, the effects of cnoidal wave and seabed characteristics, such as water depth, wave height, wave period, permeability, elastic modulus, and degree of saturation, on the cnoidal wave-induced excess pore pressure and liquefaction phenomenon are studied.

1. Introduction

It is generally known that Stokes wave is the most useful wave type when depth to wavelength ratio d/L is greater than about 1/8 or 1/10 [1]. For shallower water, cnoidal wave theory is more satisfactory, which the formula for the wave profile involves Jacobian elliptic function. The theory of cnoidal wave has not been developed and used by engineers because the theory includes Jacobian elliptic function $sn(\,)$, $cn(\,)$, which is difficult to be calculated. The object of the present paper is to fill this need.

Wiegel [2] studied the first-order cnoidal wave and gave some graphs for practical application. They showed the comparison between theory and laboratory measurement. Fenton and Gardiner-Garden [3] developed alternative methods for the calculation of elliptic integrals by using the imaginary transformation, which is simple and converges more quickly in limit corresponding to cnoidal wave. Moreover, Fenton [4] investigated cnoidal theory as well as Stokes theory and Fourier approximate methods such as the "stream function method". It was found that if the series for velocity are expressed in terms of the shallowness rather than relative wave height, then results are very much better, and justified the use of cnoidal theory even for high waves. Isobe [5] obtained the solutions of the first-order cnoidal wave in terms of power series of theta functions. The wave height changes due to shoaling, refraction, and wave set-down are calculated. Synolakis et al. [6] studied the maximum relative runup of cnoidal wave climbing up a plane beach. It was shown that the maximum relative runup of cnoidal wave is not a monotonically varying function of the normalized wavelength. Cho [7] proposed to use the Newton–Raphson method to estimate the Jacobian elliptic parameter, which is useful in generating a train of cnodial water wave. Chang et al. [8] investigated the vortex generation and evolution due to flow separation around a submerged rectangular obstacle under cnoidal wave both experimentally and numerically. Grajales [9] investigated the stability of cnoidal waves for a generalized Benney–Luke equation. They obtained numerical evidence regarding the accuracy and properties of the numerical scheme in some experiments. Ertekin et al. [10] studied the cnoidal wave by using the Irrotational Green–Naghdi equations. The problem of cnoidal waves passing over a submerged shelf was presented using GN Level I and LGN Levels I–V equations.

Similarly, based on different assumptions of the rigidity of soil skeleton and the compressibility of pore fluid, numerous theories have been developed for the wave-induced soil response and liquefaction [11–34]. Among these, Zienkiewicz et al. [13] investigated the u–p approximation model to porous flow through one-dimensional analysis, which has been first applied to the wave-induced soil response. Ishihara and Yamazaki [35] evaluated the magnitude of cyclic stress and wave-induced liquefaction on the

* Corresponding author at: State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, Shanghai 200240, China. Tel.: +86 21 3420 4833; fax: +86 21 3420 4833.
E-mail addresses: zhouxl99@hotmail.com, zhouxl@sjtu.edu.cn (X.-L. Zhou).

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basis of design storm parameters in deep water. They found that for a medium dense deposit of sand with 70% relative density, liquefaction in several storms could be 17.7 m deep in a water depth of 14 m. Zen and Yamazaki [36–38] studied the mechanism of the wave-induced liquefaction in the seabed by using the experimental and analytical results and field monitoring. The criterion of Zen and Yamazaki [36,37] can be utilized to investigate the liquefaction potential from the viewpoint of "excess pore pressure", based on one-dimensional elastic analysis. Based on Mei and Foda [14] model, Yuhi and Ishida [20] proposed an analytical solution for the wave-induced seabed response, which can directly solve the boundary value problem. Sassa and Sekiguchi [39] studied the behavior of sand seabed under fluid wave trains by using centrifuge modeling. They found that the process of liquefaction of the sand bed under progressive wave loading is characterized by a downward advancement of the liquefaction front. Jeng and Cha [22] proposed an analytical solution of dynamic response for a porous, isotropic seabed under wave loading based on "*partly dynamic*" approximation and "*fully dynamic*" approximation. Liu and Jeng [23] established a simple semi-analytical model for the random wave-induced soil response for an unsaturated seabed of finite thickness. The maximum liquefaction depth under the random waves was also examined. Ulker and Rahman [25] developed a set of generalized analytical solutions for the wave-induced response of a saturated porous seabed. Noorzad et al. [40] carried out the wave-induced liquefaction analysis by using mechanisms similar to earthquake-induced liquefaction potential of seabed sand deposits. Xu and Dong [41] presented a numerical study of liquefaction potential of a sand bed under narrow-band random waves by employing ensemble modeling techniques. They investigated the effect of random waves on excess pore pressure build-up and liquefaction processes. Zhou et al. [30] developed an analytical solution for the wave-induced seabed response in a multi-layered porous seabed by using Fourier transformation and Transmission and reflection Matrices method. Then, Zhou et al. [31,32] investigated the wave and current induced isotropic and anisotropic seabed response around a submarine pipeline. The third-order solution of wave-current interactions is used to determine the dynamic pressure acting on the seabed. Jeng [33] had developed a series of closed-form analytical solutions for the soil response in a porous seabed subjected to water wave. These solutions had been used for the prediction of the wave-induced seabed instability such as liquefaction and shear failure. Ye et al. [42] investigated the interaction between breaking wave, seabed foundation and composite breakwater. They also predicted the liquefaction potential of seabed under breaking wave loading.

To date, the wave-induced seabed response has not been fully understood because of the complicated behavior of water wave and seabed. Most previous investigations of the water wave problem have only been concerned with Airy wave and Stokes wave, only a few researchers attempted to consider cnoidal wave in shallower water. Cnoidal wave theory is more available than Stokes wave theory when Ursell number is larger than 40 in shallow water region [43]. The cnoidal theory has not been applied for practical problem because the unfamiliarity of Jacobian elliptic functions and integrals, perceived difficulty to be dealt with. The phenomenon of liquefaction in a porous seabed subjected to cnoidal water wave loading especially has not yet been clearly addressed in nearshore engineering. Therefore, it is still necessary to understand the influence of cnoidal wave and seabed interaction and liquefaction phenomenon.

The aim of this paper is to investigate the cnoidal wave-induced porous seabed response and liquefaction phenomenon in shallower water region. The Biot's partly dynamic equation (u–p model) is solved by FEM, which both displacement and pore pressure are the field variables, is adopted considering the acceleration of soil skeleton. The cnoidal function can be calculated by using Taylor series expansion and precise integration method. Then, the cnoidal wave and seabed parameters such as: water depth, wave height, permeability, elastic modulus and degree of saturation are studied. The possibility of cnoidal wave-induced liquefaction occurring in the porous seabed is also investigated.

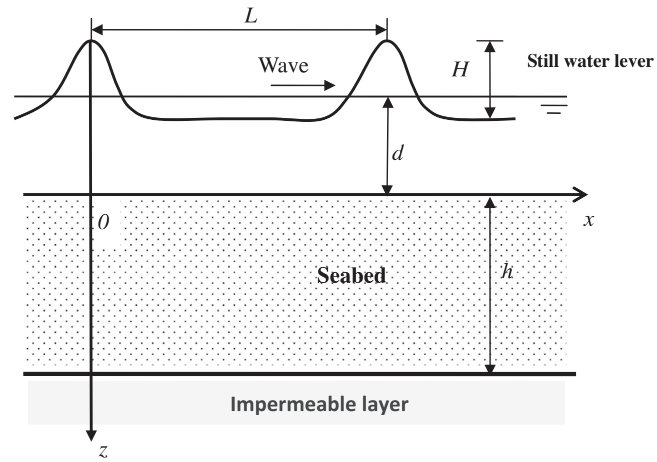

Fig. 1. Sketch of cnoidal wave and seabed.

2. Cnoidal wave and series solution

Consider a series of cnoidal wave propagation over a porous seafloor with a finite thickness above a rigid impermeable bottom, as shown in Fig. 1. Herein, the cnoidal water wave level located at $z = -d$ are traveling along the positive x-direction, and assume the vertical z-axis is downward from the surface of seabed (water–soil interface, $z = 0$) as illustrated in Fig. 1. The free water surface profile of first-order cnoidal wave can be written as [6,44]

$$\eta = H\left(\frac{1}{m^2} - 1 - \frac{E}{m^2K}\right) + Hcn^2\left[2K\left(\frac{x}{L} - \frac{t}{T}\right)\right] \quad (1)$$

where H is the wave height; $cn(\,)$ is the cnoidal function; x is the horizontal coordinate; L is the wave length, $L = 4Kmd\sqrt{d/3H}$; T is the wave period; m is the modulus; K and E are the first and second kind of complete elliptic integral, respectively and can be expressed as

$$K = \int_0^{\pi/2} \frac{1}{\sqrt{1 - m^2\ \sin^2\ \varphi}} d\varphi \quad (2)$$

$$E = \int_0^{\pi/2} \sqrt{1 - m^2\ \sin^2\ \varphi d\varphi} \quad (3)$$

Solving the cnoidal wave problem is to evaluate the Jacobian elliptic parameter m, elliptic integrals K and E and function cn.

Firstly, the formulas of Jacobian elliptic function are studied in this section. The Taylor expressions for Jacobian elliptic functions at $x = 0$ have the following forms [45]

$$sn(x, m) = x - \frac{1 + m^2}{6}x^3 + o(x^5) \quad (4)$$

$$cn(x, m) = 1 - \frac{1}{2}x^2 + (1 + 4m^2)\frac{x^4}{4!} - o(x^6) \quad (5)$$

$$dn(x, m) = 1 - m^2\frac{1}{2}x^2 + m^2(4 + m^2)\frac{x^4}{4!} + o(x^6) \quad (6)$$

The elliptic functions have also the following formulas [45]

$$sn(2x, m) = \frac{2sn(x, m)cn(x, m)dn(x, m)}{1 - m^2sn^4(x, m)} \quad (7)$$