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The initial stage of dam-break flow of two immiscible fluids. Linear analysis of global flow

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ABSTRACT

Two-dimensional impulsive flow of two immiscible fluids is studied within the potential flow theory. Initially the fluids of different depths and different densities are at rest and separated with a thin vertical plate. The plate is withdrawn suddenly and gravity-driven flow of the fluids starts. During the early stage the flow is described by the linear potential theory. Attention is paid to the motion of the interface between the fluids and the singular behaviour of the velocity field at the triple point, where the free surfaces of the fluids and the interface meet each other. The linear problem is solved by the Fourier series method. Local analysis of the flow field close to the triple point reveals that the singularity of the flow depends on the ratio of the fluid densities with a coefficient dependent on both the density ratio and the shape of the flow region. The flow velocity is also log-singular at the point where the interface meets the bottom. The intensity of this singularity depends on the density ratio. The latter singularity disappears when the densities of the fluids are equal. The Fourier series solution supplemented by the singularity analysis at the corner points resolves these initial singularities. Comparisons with solutions obtained through the boundary element method are established for validation purposes. The numerical analysis of the problem by the boundary element method is carried out and it compares quite well with the Fourier series solution. The singular flow field which leads to the jet formation at the initial instant has been observed by both methods. The problem of dam-break flow for the wet-bed case corresponds to the present problem with equal densities of the fluids. Comparisons with data available in literature are established in the case of fluids with the same density.

1. Introduction

Plane unsteady problem of the gravity-driven flow of two immiscible fluids is considered. Initially the fluids are separated with a thin vertical plate and are at rest. The fluid depths are not equal to each other. At the initial time instant, $t' = 0$, the plate is instantly removed. The flow of two different fluids driven by the gravity starts suddenly (see Fig. 1). The present study is concerned with the initial stage of the flow, during which the fluid displacements are small compared with the fluid depths. The problem is studied in the Cartesian coordinate system (x', y'). A prime stands for dimensional variables. The line $y' = 0$ corresponds to the rigid bottom. The interval, $x' = 0$, $0 < y' < H^-$, corresponds to the initial position of the interface between the fluids. Quantities corresponding to the fluid which is originally on the left of the plate with the depth H^- are denoted by the superscript minus. Quantities describing the fluid on the right of the plate with the depth H^+ are denoted by the superscript plus. Densities of the fluids are ρ^- and ρ^+, correspondingly. The ratio of the densities is denoted by $\gamma = \rho^-/\rho^+$. In the present analysis, $H^+ > H^-$ and $0 < \gamma < \infty$.

This problem was studied in the past for two identical fluids, $\gamma = 1$, and for $\gamma \neq 1$ but without the free surfaces of the fluids. The first problem is known as a wet-bed dam-break problem [1–4] and the second as a stratified mixing problem [5].

Both problems have been studied using the nonlinear shallow water equations, see Goater and Hogg [6] and Gill [7], p. 259, correspondingly. The shallow water approximation is based on the assumption that the vertical component of the flow velocity is negligible compared to the horizontal components and the pressure is hydrostatic. The stratified mixing in a dam-break problem was studied in [5] with the aim of extending the Large-Eddy Simulation (LES) method to small-scale mixing problems. A simplified configuration similar to that in Fig. 1 with $H^+ = H^-$ and rigid upper boundary was chosen to provide a test case with well defined initial and boundary conditions for a complex flow in a rectangular enclosed domain. Özgökmen et al. [5] wrote "When non-hydrostatic pressure is not modelled while nonlinearities are (i.e. in a hydrostatic model), the wave steepens unabated and may cause an artificial mixing event, depending on the turbulence closures employed. Also, the dispersion relations for internal waves differ in hydrostatic and non-hydrostatic

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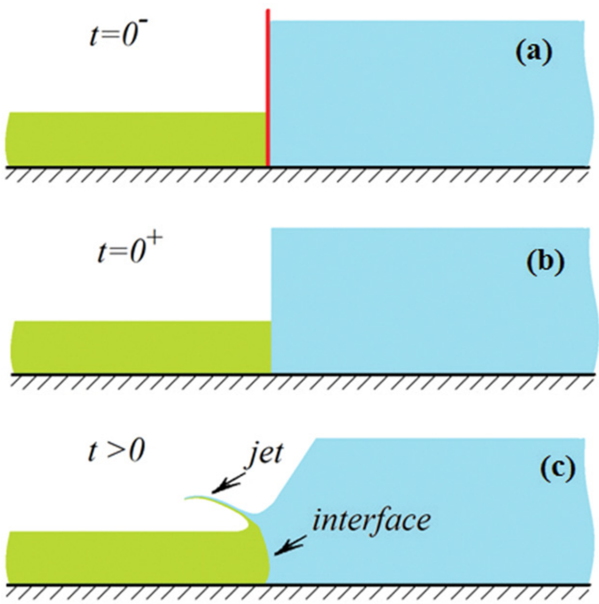

Fig. 1. Scheme of the gravity-driven flow: (a) initially the fluids are at rest and separated with a vertical plate; (b) the plate is removed and the flow starts at $t = 0^+$; (c) expected flow of two immiscible fluids with the jet consisting both fluids.

cases. Thus, hydrostatic models not only are unable to correctly represent the shape of the internal waves and their degeneration into solitary waves, but they can also fail to represent the wave-induced boundary mixing."

The problem of wet-bed dam-break flow is very challenging. Even for the simplified geometry shown in Fig. 1, the initial conditions on the free surfaces of the fluids do not match each other which gives rise to a singularity of the initial flow at the triple point, where the free surfaces and the interface meet each other. This singularity and the resulting jet formed at the corner point shortly after the plate is withdrawn were discovered by Stansby et al. [1] for the fluids of equal densities. Both experimental and numerical studies of this new phenomenon were conducted in [1]. It was shown that the initial flow is singular at the corner point and a mushroom-like jet occurs just after release. The singularity was not studied. The jet observed in the experiments was well reproduced by the boundary-element method described by Cooker et al. [8]. To start the numerical simulations the singularity was smoothed out with the initial step-like free surface being approximated by a hyperbolic tangent. The obtained solutions were used by Lind et al. [4] (see Section 4.2.2.2 in [4]) to validate the method of incompressible smoothed particle hydrodynamics. The mushroom-like jets were also observed in experiments by Janosi et al. [2]. Gomez-Gesteira et al. [3] used these experimental results to demonstrate the capabilities of the smoothed particle hydrodynamics (SPH) method to reproduce the observed shapes of the free surface after the lock release. It is seen that the solution of the dam-break problem for wet bed case was used to validate some numerical algorithms.

The present analysis combines the stratified mixing problem and the dam-break problem for wet-bed case with focus on the local flow close to the triple point. The jet formed at this point is of interest in hydrodynamics of high-speed ships, where separation of the flow occurs at the intersection of the boards with the stern plane [9]. Martinez-Legazpi [9] wrote "The difference in height between the separated stream and the free surface level immediately downstream the stern induces a transversal velocity component to the separated water mass that deflects its otherwise stream-like velocity towards the centerline of the hull. As a result, two symmetrical waves are formed that collide near the center plane of the wake." Taking advantage of the slender nature of the flow, the three-dimensional steady flow was approximated by a two-dimensional unsteady one. Then the flow close to the stern corresponds to the initial flow in a dam-break problem. Note that the latter problem differs from that studied by Stansby et al. [1] due to different initial conditions of the flow. The corner jets and resulting waves were studied experimentally, numerically and theoretically in [9].

Flow singularities at the intersection points between a liquid free surface and a solid boundary were studied intensively in the past. Initial stage of impulsive motion of a body piercing the free surface was analysed in [10,11] for vertical and horizontal impacts of a semi-submerged circular cylinder, in [12,13] for floating wedge and plate of zero draft, in [14–16] for horizontal impact of a vertical wall. It was shown that jet is developed at the intersection point with the jet strength being dependent on the angle between the body surface and the free surface of the liquid and the direction of the body motion. The flow in the jet region is nonlinear and self-similar during the early stage. Little is known about free-surface flows starting suddenly but with zero velocity. Initial flows generated by an accelerating plate were studied in [15] for vertical plate and in [16] for inclined plate. The papers [15,16] are relevant to the present study, where the flow also starts from rest with zero initial velocity. It is interesting to note that the flow caused by uniformly accelerated vertical plate in [15] and the dam-break flow studied in [17] are locally similar close to the corresponding intersection points. In the present paper, the corresponding intersection point is at the bottom, where the interface between two fluids meets the solid boundary. However, there is another point, the triple point, which makes the problem more complicated than the problems mentioned above.

We assume a jet formed at the triple point. The shape of the jet and its structure can be different from that shown in Fig. 1c. One may expect a mushroom-like jet similar to that observed in [1] for fluids of equal densities. We expect that locally the jet flow is self-similar for small times after the flow starts. The structure of the jet should be investigated. In particular, we need to know portions of the fluids in the jet. This can explain the gravity-driven mixing of fluids. We expect that the local flow of immiscible fluids near the triple point is governed by the global flow. However the global flow depends weakly on the local one and can be approximately determined on its own during the early stage. Next, local flow can be recovered in properly stretched local coordinates and matched to the global flow. In this paper, we are concerned with the leading order global flow and matching conditions for the local region near the triple point. A formal small parameter is introduced to specify that only the initial stage of the flow is considered. The leading order global flow is described by a linear boundary value problem, which is obtained by linearisation of the boundary conditions and imposing them on the initial undisturbed positions of the boundaries of the fluids. The linear problem is solved by the Fourier series method and by the method of boundary element. The obtained velocity field is singular at both the triple point and intersection point between the interface and the flat bottom. The singularity of the flow is recovered with the help of local analysis and the Fourier method. The local analysis of the global flow at the triple point provides the order of the singularity and the solution obtained by the Fourier method provides the coefficient of the singularity. The boundary element method is used to verify the solution by the Fourier series method for different ratios of the fluid densities and different depths of the fluids.

The formulation of the small-time linearised problem is given in Section 2. In terms of uniformly valid small-time asymptotic solution of the fully nonlinear problem, in this paper, we restrict ourselves to the leading order outer solution. The numerical solution of this leading-order problem is obtained in Section 3 by the Fourier method. Local behaviour of the solution close to the triple point is studied in Section 4. The coefficient of the singularity is determined by using the Fourier series solution. The boundary element method is applied