Applied Ocean Research 42 (2013) 144–154

Contents lists available at SciVerse ScienceDirect

Applied Ocean Research

journal homepage: www.elsevier.com/locate/apor

Boussinesq modelling of solitary wave and N-wave runup on coast

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ARTICLE INFO

Article history:
Received 17 December 2012
Received in revised form 28 May 2013
Accepted 30 May 2013

Keywords:
Runup
Solitary waves
N-waves
Total variation diminishing
Boussinesq equations

ABSTRACT

A total variation diminishing Lax–Wendroff scheme has been applied to numerically solve the Boussinesq-type equations. The runup processes on a vertical wall and on a uniform slope by various waves, including solitary waves, leading-depression N-waves and leading-elevation N-waves, have been investigated using the developed numerical model. The results agree well with the runup laws derived analytically by other researchers for non-breaking waves. The predictions with respect to breaking solitary waves generally follow the empirical runup relationship established from laboratory experiments, although some degree of over-prediction on the runup heights has been manifested. Such an over-prediction can be attributed to the exaggeration of the short waves in the front of the breaking waves. The study revealed that the leading-depression N-wave produced a higher runup than the solitary wave of the same amplitude, whereas the leading-elevation N-wave produced a slightly lower runup than the solitary wave of the same amplitude. For the runup on a vertical wall, this trend becomes prominent when the wave height-to-depth ratio exceeds 0.01. For the runup on a slope, this trend is prominent before the strong wave breaking occurs.

1. Introduction

With the population and economic activities continuously being concentrated towards the coastal area, it is increasingly important to protect the coastal infrastructure from wave impact and inundation in extreme events, which are anticipated to occur more frequently. Even when the wave crest in the open sea is lower than the ground level onshore, risk still exists because of the wave runup, which may significantly amplifies the free surface elevation.

Wave runup is a classical hydrodynamic problem. In the canonical configuration, a periodic or single wave propagates over a constant-depth region and then climbs up a plane beach of a constant slope, as shown in Fig. 1. The uniform slope of angle β represents an idealised beach connecting the flat offshore region and land. In the figure, h is the undisturbed depth, h_0 is the undisturbed depth in the flat region, η is the water surface position above the still water level, A_0 is the amplitude of the incident wave. The runup, R_m, is defined as the maximum vertical elevation above the still water level reached by water on the slope. During shoaling from the deep to shallow waters, the asymmetry of the wave profiles increases and the wave height grows. Wave breaking may occur, if the incident waves are high and the slope is gentle.

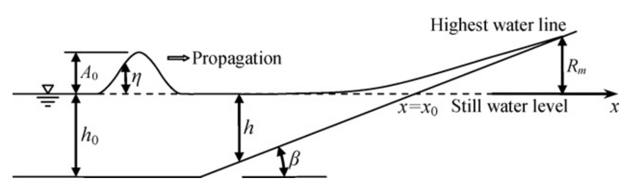

Fig. 1. Definition sketch of the solitary wave propagation and runup on a slope.

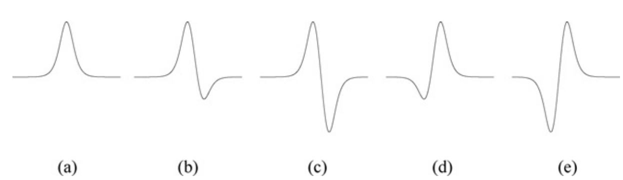

Fig. 2. Profiles of the right-travelling solitary wave and N-waves: (a) solitary, (b) LDN with $\mu = 0.5$, (c) LDN with $\mu = 1.0$, (d) LEN with $\mu = 0.5$, and (e) LEN with $\mu = 1.0$.

The theory of the nonlinear long waves has advanced significantly with the development of "soliton theory", which began with the Korteweg-de Vries (KdV) equation. A lot of analytical, numerical and experimental studies on solitary waves have been connected to tsunamis (*e.g.*, [1–3]). However, Tadepalli and Synolakis [4] argued that the main tsunami wave was often preceded by a depression, and thus introduced the concept of N-waves in order to achieve better geophysical relevancy. With the accumulated evidence from field observations, it is generally acknowledged that the first couple of

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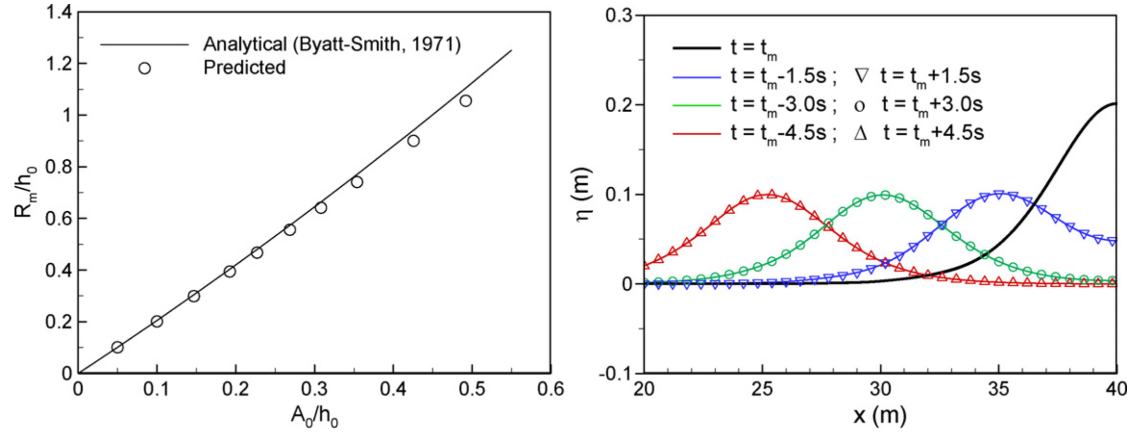
0141-1187/$ - see front matter
http://dx.doi.org/10.1016/j.apor.2013.05.008



(a) Runup heights for different wave amplitudes (b) Wave profiles at different times

Fig. 3. Verification of solitary runup on a vertical wall.

waves in the front of tsunamis approach beaches like either leading-depression N-waves (LDN) or leading-elevation N-waves (LEN), depending on the rupture properties in the subduction zone. Madsen and Schäffer [5] questioned the validity of the classical solitary wave paradigm in tsunami studies, from the perspective that solitary waves have drastically different time and length scales in comparison with geophysical tsunamis. They cast doubt on whether the classical KdV or solitary wave theory is applicable to representing the main features of real tsunami events.

For non-breaking solitary waves and N-waves, analytical solutions have been acquired concerning their runup processes. Synolakis [1] derived the first runup formula for incoming solitary waves. Tadepalli and Synolakis [4] gave one formulation of N-waves together with their runup solutions. Madsen and Schäffer [5] proposed another form of N-waves, whose wavelength is not linked to the wave nonlinearity as in the solitary wave theory, and they also obtained the analytical runup formulae for incoming single waves and LDN. By applying the relationship dictated by the solitary wave theory, they recovered the solution of Synolakis [1]. No analytical solutions exist for breaking waves. More recently, numerical long-wave models have been widely employed in studying the runup process. Both the shallow water equations (*e.g.*, [2,3,6]) and Bounssinesq equations (*e.g.*, [7–9]) have been widely used.

It is not the purpose of this paper to justify which type of waves is more suitable for representing tsunamis. We applied a newly developed shock-capturing Bounssinesq-type model to simulate the runups of various waveforms on a vertical wall and on a uniform slope, without making association with tsunami phenomena. The solitary waves, LDN and LEN of different heights were simulated. We examined the way that the waves changed their shapes during runup, and focused on the maximum runup height, which is an important parameter for assessing the wave impact on beaches, whether it is the tsunami wave, storm surge, rogue wave or any other type of large coastal swells. The runup heights produced by different types of waves were compared.

2. Mathematical model

2.1. Governing equations

If the wavelength is over twenty times larger than the water depth, then the wave can be classified to be shallow, for which the frequency dispersion can be totally ignored and the pressure distribution can be assumed to be hydrostatic. Boussinesq-type equations extend the shallow-water theory by incorporating the non-hydrostatic pressure distribution, which arises from the vertical acceleration of water particles and can be linked to the curvature of the water surface profile. Although the standard Boussinesq equations include terms that model dispersion [10], the range of their applicability is not significantly greater than the shallow water equations. From a linear dispersion point of view, the depth-to-wavelength ratio has to be less than 0.15. Over the years, various modified Boussinesq-type equations have been proposed in the literature, such as Witting [11], Madsen and Sørensen [12], and Nwogu [13]. These models have extended the standard Boussinesq equations to be applicable to intermediate-depth water with the depth-to-wavelength ratio being up to 0.5. In order to achieve this, Madsen and Sørensen [12] introduced additional third-order terms into the standard Boussinesq equations to obtain a system of equations with a dispersion relation closely matching the linear wave theory. Nwogu [13] took another approach by deriving equations at an arbitrary depth and making connections between the depth of the formulation and the dispersion characteristics. Through numerical experiments, Shiach [14] found that the Madsen and Sørensen's Boussinesq-type equations provided marginally better results than the Nwogu's formulation in terms of wave runup.

The present study adopted the one-dimensional Boussinesq-type equations of Madsen and Sørensen [12], which take into account the effects of uneven bed elevations. The continuity equation and momentum equation are:

$$\eta_t + q_x = 0 \tag{1}$$

$$\begin{aligned} q_t + \left(\frac{q^2}{d} + \frac{g\eta^2}{2} + gh\eta\right)_x &= g\eta h_x + \left(B + \frac{1}{3}\right) h^2 q_{xxt} \\ &+ Bgh^3 \eta_{xxx} + hh_x \left(\frac{1}{3} q_{xt} + 2Bgh\eta_{xx}\right) \end{aligned} \tag{2}$$

where x is the horizontal coordinate, t is the time, the subscripts t and x denote the partial derivatives with respect to time and space, respectively, the two unknown variables $\eta(x, t)$ and $q(x, t)$ are the free-surface displacement above the still water level and the volumetric discharge per unit width, respectively, $h(x)$ is the undisturbed water depth, $d(x, t) = \eta(x, t) + h(x)$ is the total water depth, and g is the acceleration due to gravity. The last three terms on the right hand side of Eq. (2) are the Boussinesq dispersive terms, which account for the non-hydrostatic effect on pressure. B is the Boussinesq dispersion enhancement coefficient, which can be tuned to match the frequency dispersion of real water waves. $B = 0$ implies no improvement, and the system reverts back to the classical Boussinesq equations originally