Applied Ocean Research 47 (2014) 73–84

Contents lists available at ScienceDirect

Applied Ocean Research

journal homepage: www.elsevier.com/locate/apor

Water wave scattering by an elastic plate floating in an ocean with a porous bed

Paramita Maiti [a], B.N. Mandal [b,*]

[a] Department of Mathematics, Jadavpur University, Kolkata 700032, India
[b] NASI Senior Scientist, Physics and Applied Mathematics Unit, Indian Statistical Institute, 203, B.T. Road, Kolkata 700 108, India

ARTICLE INFO

Article history:
Received 8 August 2013
Received in revised form 18 March 2014
Accepted 19 March 2014
Available online 8 May 2014

Keywords:
Water wave
Elastic plate
Eigenfunction expansion
Porous bed
Reflection and transmission coefficients

ABSTRACT

The problem of water wave scattering by a thin horizontal elastic plate (semi-infinite as well as finite) floating on an ocean of uniform finite depth in which the ocean bed is composed of porous material of a specific type is analyzed. The method of eigenfunction expansion is used in the mathematical analysis and the quantities of physical interest, namely the reflection and transmission coefficients, are obtained. Numerical estimates for these coefficients are obtained for different values of the parameter describing the porosity of the ocean bed and for different edge conditions of the elastic plate. The edge conditions considered here involve (i) a free edge, (ii) a simply supported edge and (iii) a built-in edge. From the numerical results it is observed that for free edge condition, the porosity of the ocean bed has little effect on the reflection and transmission coefficient for both the cases of semi-infinite and finite elastic plate. The energy identity related to reflection and transmission coefficients in a porous bed is derived and is used as a partial check on the correctness of the numerical results for the semi-infinite elastic plate.

1. Introduction

In coastal areas, porous structures are widely used as breakwaters to protect harbours, inlets and beaches from wave action. The submerged structure is usually adopted with a porous material that has the function of ecological restoration in the coastal area. Physically, the porous structure is able to absorb and dissipate the wave energy when the incident wave transmits over it. Silva et al. [1], Zhu [2] considered water-wave reflection and/or transmission problems where a porous medium was assumed to lie on a seabed of varying quiescent depth. Tsai et al. [3] investigated the wave transmission over a submerged permeable breakwater on a porous sloping seabed. Yu and Chwang [4] investigated the wave motion through a two-layer porous structure. They presented numerical results for the special cases where the two layers are of the same material properties or where porous medium exists only in the lower layer. Many theoretical as well as numerical investigations were also carried out for the permeable or impermeable submerged breakwaters in the water wave literature, e.g. Losada et al. [5], Liu et al. [6], Hur and Mizutani [7] and others.

In the present paper we have considered the interaction of a plane incident surface wave with a semi-infinite elastic plate [8,9] and with a finite elastic plate [10,11] floating or submerged in ocean regarded as an inviscid fluid of uniform finite depth with a porous bed. Meylan and Squire [10] considered the interaction of water wave with a finite elastic plate in both infinite and finite depth water while Hassan et al. [11] considered only finite depth water. Here we consider a special type of porous bottom [12] in which the motion of the fluid inside the porous bed is not analyzed and it is assumed that the fluid motions are such that the resulting boundary condition on the sea-bed used in this paper holds good and depends on a known parameter G which has a dimension of $(\text{length})^{-1}$, called the porosity parameter. We consider the value of this porosity parameter G to be only real. Due to the percolation at the fluid-porous interface, the porosity parameter G can be chosen to be real in which only flow resistance is considered by neglecting the inertial terms. It may be mentioned here that for existence of progressive waves on the surface of the open water region and also on the thin elastic plate floating on water, it is necessary that each of the corresponding dispersion equations ((A.1) and (A.4) in Appendix A) must possess a unique real positive root. This is possible only if G is real. However, if G is chosen to be complex, then it can be shown that each of the dispersion equations does not have any real positive root so that there does not exist any progressives waves on the upper surface. There will be damping of wave energy and no wave can propagate on the upper surface of water in this situation.

Various approaches for the study of wave interaction with an elastic plate were summarised by Teng et al. [8] and Fox and Squire [9]. Fox and Squire [9] used the eigenfunction expansion method

* Corresponding author. Tel.: +91 335753034; fax: +91 3325773026
E-mail address: biren@isical.ac.in (B.N. Mandal).

0141-1187/$ – see front matter © 2014 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.apor.2014.03.006

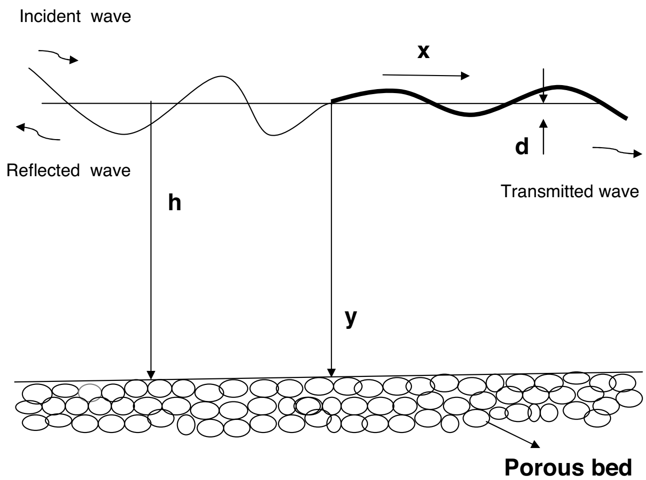

Fig. 1. Sketch of definition for semi-infinite elastic plate.

to study wave interaction with a floating semi-infinite thin elastic plate and they observed that the eigenfunctions are not orthogonal with respect to the conventional inner product. Sahoo et al. [13], Xu and Lu [14] defined a new inner product in which the original eigenfunctions became orthogonal. Bhattacharjee and Sahoo [15], Xu and Lu [16] further applied this method for the case of a two-layer fluid.

Three different types of edge conditions, namely (i) a free edge, (ii) a simply-supported edge and (iii) a built-in edge, are considered. It may be noted that for a free edge, the shear force and the bending moment of the plate vanish at the edge. However, artificial structures are usually kept fixed or moored at the edge by ropes, anchors, tension cables or piles. In such cases the free edge condition is to be replaced by the simply supported edge condition or the built-in edge condition as per reality. It may be noted that for the simply supported edge condition, the deflection and the bending moment are assumed to vanish, whereas for the built-in edge condition, the deflection and the slope of deflection will vanish. We use here the usual inner product and the importance of the present method is that the vertical eigenfunctions due to this porous bed profile are orthogonal in the open water region which is the same as in Xu and Lu [14]. The energy balance equation [17,18] for the semi-infinite elastic plate in porous bed related to reflection and transmission coefficients are derived by using Green's identity in Appendix B. While computing these coefficients numerically for a particular wave number and for various other parameters corresponding to the semi-infinite elastic plate, it has been checked that these satisfy the appropriate energy identity.

2. Wave interaction with a semi-infinite elastic plate

2.1. Mathematical formulation

The problem under consideration is two-dimensional in nature. The Cartesian coordinates Ox, Oy are chosen in such a way that y-axis is measured positive vertically downwards and the plane $y=0$ coincides with the undisturbed upper surface. A floating semi-infinite elastic plate of very small thickness d occupies the position $y=0$, $0<x<\infty$ while the ocean at rest has the domain $-\infty<x<\infty$, $0\le y\le h$ (cf. Fig. 1). The ocean bottom is composed of some specific kind of porous materials of the porosity parameter G which has a dimension of inverse of length and is taken to be real. It is assumed that the ocean water is inviscid and incompressible. Under the assumptions that the motion is irrotational and simple harmonic in time with circular frequency ω, the velocity potential describing the motion in the fluid is represented by $\mathrm{Re}\{\phi(x,y)e^{i\omega t}\}$, where ϕ satisfies

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0 \quad \text{in} \quad 0\le y\le h, -\infty<x<\infty. \tag{1}$$

The bottom condition is

$$\phi_y - G\phi = 0 \quad \text{on} \quad y=h, \quad -\infty<x<\infty, \tag{2}$$

where G is the porosity parameter of the ocean bottom (cf. Martha et al. [12]).

We assume that any particle which is once between the elastic plate and the water surface remains there. Under this assumption, the linearised kinematic condition is given by

$$\phi_y = -i\omega\eta \quad \text{on} \quad y=0, \quad 0<x<\infty, \tag{3}$$

$Re(\eta e^{-i\omega t})$ denoting the depression of the thin elastic plate below its rest position for $x>0$ and of the free surface for $x<0$.

The conditions on the upper surface $y=0$ are given by

$$\omega^2\phi + g\frac{\partial\phi}{\partial y}=0 \quad \text{on} \quad y=0, \quad -\infty<x<0, \tag{4a}$$

$$\rho\omega^2\phi + \left(EI\frac{\partial^4}{\partial x^4} - m_e\omega^2 + \rho g\right)\frac{\partial\phi}{\partial y}=0 \quad \text{on} \quad y=0, \quad 0<x<\infty, \tag{4b}$$

where $EI=(Ed^3/12(1-\nu^2))$ is the flexural rigidity of the plate, E being the effective Young's modulus of the elastic plate, ν being the Poisson's ratio, $m_e=\rho_e d$, ρ_e is the density of the elastic plate, ρ is the fluid density and g is the gravitational acceleration.

The progressive gravity waves propagating at each of the free surface and elastic plate regions can be expressed by the potential function

$$\phi(x,y) = e^{\pm ikx}\frac{k\cosh k(h-y) - G\sinh k(h-y)}{k\cosh kh - G\sinh kh} \tag{5}$$

where k is real and positive, and for waves in the open water and the region covered by the elastic plate k satisfies respectively the equations

$$\omega^2(k-G\tanh kh) - kg(k\tanh kh - G)=0, \tag{6a}$$

and

$$\rho\omega^2(G\tanh kh - k) + (EIk^4 - m_e\omega^2 + \rho g)(k^2\tanh kh - kG)=0. \tag{6b}$$

Eqs. 6a and (6b) are the dispersion equations for the open water region and elastic plate covered region respectively. Determination of the roots of these two equations are given in Appendix A.

Further it is assumed that at the edge $(0^+, 0)$ where the plate meets the free surface, one of the following edge conditions are satisfied at $(0^+, 0)$ [19].

(i)Free edge: There is zero bending moment and zero shear force at the free edge of the plate so that $\partial^2\eta/\partial x^2=0$ and $\partial^3\eta/\partial x^3=0$ at $(0^+, 0)$. Using condition (3) and eliminating η we derive that for a free edge plate, the edge conditions are

$$\frac{\partial^3\phi}{\partial x^2\partial y}(+0,0)=0, \qquad \frac{\partial^4\phi}{\partial x^3\partial y}(+0,0)=0. \tag{7}$$

(ii)Simply-supported edge: At a simply-supported edge the displacement is zero and there is no ending moment so that η and $\partial^2\eta/\partial x^2$ vanish at $(0^+, 0)$. Using condition (3) and eliminating η we derive that for a simply supported edge plate, the edge conditions are

$$\frac{\partial\phi}{\partial y}(+0,0)=0, \qquad \frac{\partial^3\phi}{\partial x^2\partial y}(+0,0)=0. \tag{8}$$