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Coupled analysis of nonlinear sloshing and ship motions

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ABSTRACT

A coupled numerical model considering nonlinear sloshing flows and the linear ship motions has been developed based on a boundary element method. Hydrodynamic performances of a tank containing internal fluid under regular wave excitations in sway are investigated by the present time-domain simulation model and comparative model tests. The numerical model features well the hydrodynamic performance of a tank and its internal sloshing flows obtained from the experiments. In particular, the numerical simulations of the strong nonlinear sloshing flows at the natural frequency have been validated. The influence of the excitation wave height and wave frequency on ship motions and internal sloshing has been investigated. The magnitude of the internal sloshing increases nonlinearly as the wave excitation increases. It is observed that the asymmetry of the internal sloshing relative to still water surface becomes more pronounced at higher wave excitation. The internal sloshing-induced wave elevation is found to be amplitude-modulated. The frequency of the amplitude modulation envelope is determined by the difference between the incident wave frequency and the natural frequency of the internal sloshing. Furthermore, the coupling mechanism between ship motions and internal sloshing is discussed.

1. Introduction

With growing demands for clean energy, liquefied natural gas (LNG) attracts more and more attentions, making the exploitation of the gas fields which are located in deep waters becomes increasingly attractive and desirable. Deep waters together with the remote locations from onshore infrastructures are impeding the exploitation of these offshore gas fields. To overcome those difficulties, floating liquefied natural gas (FLNG), which consists of a ship-shaped floating hull equipped with liquefaction plants and LNG storage tanks, has been proposed recently. This new type of platform would be subjected to complex sea states which may induce severe motion responses. The wave induced motions would result in violent liquid sloshing inside the tanks disposed on the vessel. In return, the violent sloshing of internal liquid results in highly localized impact pressures on the side walls, which may cause structural damages and may even induce sufficient moment to capsize the vessel. The coupling between global motion responses and internal sloshing is quite complex. Therefore, establishing a systematic procedure to examine the coupling mechanism between ship motions and internal sloshing flows is essential for FLNG design and operation.

There have been several studies reported on the nonlinear sloshing flows inside a tank under forced excitations in translation or rotation. Faltinsen [1] carried out numerical simulations through a nonlinear model using boundary integral technique subjected to forced sway harmonic oscillations. Wu et al. [2] presented a three-dimensional model based on finite element method for the sloshing flows inside a rectangular tank under translational excitations. Akyildiz and Unal [3] carried out a series of model tests on a rectangular excited in pitch with systematically changed amplitudes to assess the sensitivity of sloshing loads on the walls. Wu et al. [4] conducted both numerical and experimental studies on forced sloshing flows with baffle in the tank, to examine the effects of baffles on resonant frequency. However, the above studies primarily focus on the nonlinear sloshing flows due to forced motions. In order to understand the coupling between the FLNG motions and the internal sloshing, further study is warranted to assess this complex coupling phenomenon.

Based on the assumption of linear sloshing flows inside the tanks, numerical simulations on the coupled problem between ship motions and internal sloshing have been conducted in frequency domain by Molin et al. [5] and Newman [6]. To incorporate the mooring forces during LNG offloading, time-domain ship motion analysis was conducted by Park et al. [7]. Gou et al. [8] conducted both frequency- and time-domain analyses of the linear ship motions and sloshing. It is found that the linear assumption is acceptable with reasonable accuracy for the prediction of global motion response. To evaluate the characteristics of the nonlinear sloshing flows, simulations in time domain based on computational fluid dynamics (CFD) were conducted [9,10]. In their studies, the linear flow is assumed for the external waves and the internal sloshing is simulated based on a CFD scheme with a finite difference method. Based on the finite element method for the simulation of nonlinear sloshing, Mitra et al. [11] went one step further to carry out numerical simulations in time domain whereas the nonlinear ship motion is assumed. In addition to the numerical studies, several experimental studies have been reported. Early experimental studies can be traced back to the report by Mikelis et al. [12], in which the coupled problem was investigated on a carrier ship. Francescutto and Contento [13] conducted an experiment to investigate the coupling effect between the roll motion of a ship and the sloshing flow in a floodable compartment, with the tank subjected to a beam sea condition. Later, Rognebakke and Faltinsen [14] carried out a series of model tests on a rectangular tank excited in sway motion by regular waves. The tank model is ballasted in different filling conditions in their studies. Similar studies have also been studied by Nasar et al. [15,16] and Lee et al. [17]. Molin et al. [5] conducted an experiment on a barge with a partially filled water tank on deck under the combined excitations of sway, heave and roll. Nam and Kim [18] carried out a series of model tests to investigate the effects of sloshing on the motion responses of FLNG. In their study, it is found that the coupling effects do not always result in the

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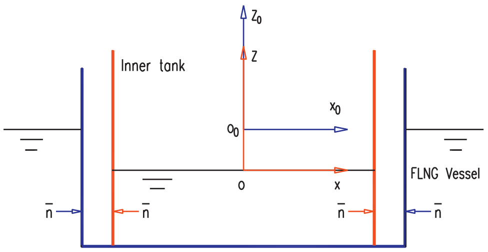

Fig. 1. The coordinate systems.

increase of sloshing-induced pressure and that the increase or decrease of pressure is dependent on resonant condition. As a summary of the literature, method based on linear assumption fails to incorporate the strong nonlinearities of internal sloshing, while the CFD method is quite time consuming and experimental technique is expensive. Thus, establishing a time-saving numerical model which is capable of considering the strong nonlinear sloshing flows is necessary and of great value.

Previous studies have shown that both sway [14] and roll [19,20] motions coupling with internal sloshing are of significant importance. In this study, we focus on the coupling between sway motion and internal sloshing. In particular, an FLNG section under the excitation of regular waves in sway has been selected as the reference. The coupling between roll motion and internal sloshing will be studied in the future. Aimed at predicting the coupled responses between sway motion and nonlinear internal sloshing, a coupled numerical model in two dimensions (2D) is proposed. To validate the numerical model, a set of experiments have been conducted. The simulated results show good agreement with those of experimental data, in both ship motions and strong nonlinear sloshing flows at the natural frequency. Based on the validated numerical model, attempts have been made to examine the influence of the wave height and the wave frequency on both ship motions and internal sloshing flows. Some interesting characteristics of the nonlinear sloshing flows have been observed. In addition, careful discussion has been made on the coupling mechanism between ship motions and internal sloshing.

2. Mathematical formulation

The numerical model developed considering both sway and heave motions is capable of dealing with any single degree-of-freedom motion (sway or heave) and the coupled motions of sway and heave. In the numerical simulation, two coordinate systems are defined: the first, $o_0x_0z_0$ is space-fixed, and the other, oxz is ship-fixed. The origin of the former system is at the center of the FLNG vessel at the external free surface, while the origin of the latter system is at the center of the FLNG vessel at the internal free surface. The definitions of these two systems are shown in Fig. 1. In this section, the mathematical formulation will be described in both sway and heave motions. The single sway motions coupled with internal sloshing can be easily obtained by letting the vertical displacement of the vessel as constant.

2.1. Modeling of nonlinear sloshing

The displacement of the FLNG vessel can be defined as the follows:

$$\vec{X}_f = [x_f(t), z_f(t)]. \tag{1}$$

Consequently, the motion velocity and acceleration of the FLNG vessel can be expressed as:

$$\vec{U} = \frac{d\vec{X}_f}{dt} = \left[\frac{dx_f}{dt}, \frac{dz_f}{dt}\right] = [u, \omega], \tag{2}$$

$$\vec{a} = \frac{d\vec{U}}{dt} = [a_x, a_z]. \tag{3}$$

2.1.1. In the space-fixed coordinate system

In potential flow theory, the fluid is incompressible and inviscid. The fluid flow is assumed to be irrotational. The fluid motion can therefore be described by a velocity potential ϕ, which satisfies the Laplace equation in the fluid domain:

$$\nabla^2\phi = 0. \tag{4}$$

On the side walls of the inner tank, the potential should satisfy:

$$\frac{\partial\phi}{\partial n} = \vec{U}\cdot\vec{n}, \tag{5}$$

where, $\vec{U}$ has been defined in Eq. (2) and $\vec{n}$ is outwards normal to the tank walls, as shown in Fig. 1.

On the free surface $z_0 = \eta_0(x_0, t)$, the dynamic boundary condition in the space-fixed coordinate system can be written as:

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}\nabla\phi\cdot\nabla\phi + g\eta_0 = 0. \tag{6}$$

The kinematic boundary condition on the free surface can be written as:

$$\frac{\partial\eta_0}{\partial t} + \frac{\partial\phi}{\partial x_0}\cdot\frac{\partial\eta_0}{\partial x_0} - \frac{\partial\phi}{\partial z_0} = 0. \tag{7}$$

The initial conditions of the system are usually given as:

$$\phi(x_0, z_0, 0) = 0, \tag{8}$$

$$\eta_0(x_0, 0) = 0. \tag{9}$$

The pressure can be written as:

$$-\frac{p}{\rho} = \left(\frac{\partial\phi}{\partial t} + \frac{1}{2}\nabla\phi\cdot\nabla\phi + gz_f\right)_{x_0z_0} \tag{10}$$

2.1.2. In the ship-fixed coordinate system

To facilitate the computation, the equations from the space-fixed coordinate system are translated to the ship-fixed coordinate system by the translating relationship between the two systems:

$$\nabla_{x_0z_0} = \nabla_{xz}, \tag{11}$$

$$\left(\frac{\partial}{\partial t}\right)_{x_0z_0} = \left(\frac{\partial}{\partial t} - \frac{d\vec{X}_f}{dt}\cdot\nabla\right)_{xz}, \tag{12}$$

the dynamic and kinematic boundary conditions (Eqs. (6) and (7)) can be written in the ship-fixed coordinate system as:

$$\frac{\partial\phi}{\partial t} - \frac{d\vec{X}_f}{dt}\cdot\nabla\phi + \frac{1}{2}\nabla\phi\cdot\nabla\phi + g(\eta + z_f) = 0, \tag{13}$$

$$\frac{\partial\eta}{\partial t} + \left(\frac{\partial\phi}{\partial x} - \frac{dx_f}{dt}\right)\cdot\frac{\partial\eta}{\partial x} - \frac{\partial\phi}{\partial z} + \frac{dz_f}{dt} = 0, \tag{14}$$

on the free surface $z = \eta(x, t)$, where $\eta = \eta_0 - z_f$ is the free surface elevation in the ship-fixed coordinate system.

Decompose the velocity potential ϕ into the following form:

$$\phi = \varphi + xu + z\omega, \tag{15}$$

where, u and ω represent the components of the motion velocity of the FLNG vessel in x and z directions, as shown in Eq. (2).

Substituting the Eq. (15) into Eqs. (4), (5), (13) and (14), we obtain the equations for the boundary value problem of the nonlinear sloshing in ship-fixed coordinate system:

$$\nabla^2\varphi = 0 \quad \text{in the fluid domain}, \tag{16}$$

$$\frac{\partial\varphi}{\partial n} = 0 \quad \text{on the side walls}, \tag{17}$$

$$\frac{\partial\varphi}{\partial t} + x\frac{du}{dt} + z\frac{d\omega}{dt} + \frac{1}{2}\nabla\varphi\cdot\nabla\varphi + g\eta = 0 \quad \text{on the free surface}, \tag{18}$$

$$\frac{\partial\eta}{\partial t} + \frac{\partial\varphi}{\partial x}\cdot\frac{\partial\eta}{\partial x} - \frac{\partial\varphi}{\partial z} = 0 \quad \text{on the free surface}. \tag{19}$$