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Second-order resonance among an array of two rows of vertical circular cylinders

H. Kagemoto [a,*], M. Murai [b], T. Fujii [a]

[a] The University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa-city, Chiba 277-8561, Japan
[b] Yokohama National University, 79-1 Tokiwadai, Hodogaya-ku, Yokohama-city, Kanagawa 240-8501, Japan

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ABSTRACT

Experimental and theoretical works are conducted on the first-order and second-order wave interactions among an array of two rows of vertical circular cylinders.

We show that both the first-order and the second-order resonance of dynamic free-surface displacement could in some cases be quite relevant among an array of two rows of vertical cylinders not only in theory but also in real phenomena. In some cases, on the other hand, we show that large free-surface displacement predicted theoretically is quite attenuated in reality probably due to viscous dissipation.

1. Introduction

Wave trapping in an array of floating bodies is one of the hot subjects of the current research works in the field of floating-body dynamics in waves. It is a scientifically interesting phenomenon, but also it has practically important implications in that wave trapping could entail large free-surface displacement and/or large forces on the bodies.

Other than first-order wave trapping, some of the recent works are focusing on the second-order wave trapping, e.g. [1–5]. The floating structures subjected to these works are mostly composed of 2–4 cylinders and they all conclude that second-order free-surface displacement could sometimes be relevant and could not be neglected in the design of corresponding structures such as TLPs (tension-leg platforms). Some of the above-mentioned works compare their theory with experimental results [1,2], but they do not necessarily refer to the results from the viewpoint of wave trapping.

In the present study, we show that both the first-order and the second-order resonance of dynamic free-surface displacement, or, in other words, wave trapping could in some cases be quite relevant among an array of two rows of vertical cylinders not only in theory but also in real phenomena. In some cases, on the other hand, we show that large free-surface displacement predicted in theory is quite attenuated in reality probably due to viscous dissipation.

First-order free-surface dynamics among an array of large number of cylinders lined up in a single row were investigated theoretically [6] and experimentally [7]. Maniar and Newman [6] have shown that a near-trapping of waves among the array could take place even when the cylinders are arrayed in one line not in the direction perpendicular to that of incident waves but along the direction of incident waves. Kagemoto et al. [7] carried out experiments in which 50 cylinders were arrayed in one line along the direction of incident waves to see if the near-trapping really takes place. They found that large free-surface displacement due to the near-trapping of waves predicted by the linear theory is significantly attenuated in reality, probably because of the viscous effects. Kashiwagi and Ohwatari [8], on the other hand, carried out experiments on the free-surface displacements around an array of 4 cylinders arranged in one line along the incident-wave direction. They decomposed the measured free-surface displacements into first-order and second-order components and compared them with theoretical predictions, although the contribution of the second-order velocity potential to the second-order free-surface displacements was not accounted for in the theoretical calculations. They showed that the measured second-order component waves were markedly different from the theoretical predictions, which accounted for the quadratic products of first-order quantities only. This fact, they say, implies that the contribution of the second-order velocity potentials to the second-order wave components is significant.

With these works in mind, the present work deals with free-surface displacements up to second-order among an array composed of two rows of cylinders. As will be shown later in this paper, when the array consists of more than one row, there exist

* Corresponding author. Tel.: +81 471364115; fax: +81 471364115.
E-mail addresses: kagemoto@k.u-tokyo.ac.jp (H. Kagemoto), m-murai@ynu.ac.jp (M. Murai), fujii@geel.k.u-tokyo.ac.jp (T. Fujii).

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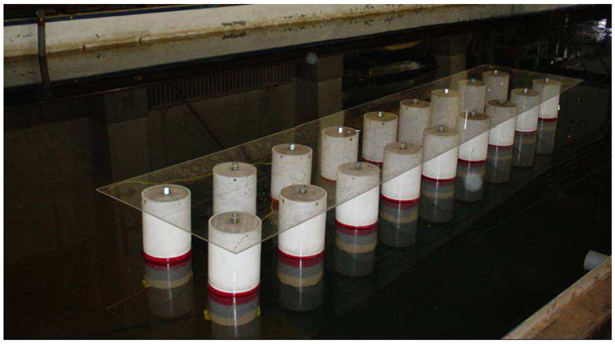

Fig. 1. The model structure supported on 2 × 9 cylinders used for the experiments.

Table 1
Particulars of the experiment.

Radius of a cylinder	$a = 0.0825$ m
Draft of a cylinder	$d = 0.215$ m
Center-to-center distance between adjacent cylinders	$L = 0.330$ m ($= 4a$)
Water depth	$H = 3.5$ m
Incident angle of incident waves	0 degree

more than one relevant free-surface resonant modes. The experimental results on the free-surface displacements are compared with theoretical calculations up to second order while taking the effect of second-order velocity potentials into account although in an approximate manner.

A floating structure supported on an array of two rows of vertical cylinders dealt with in the present study can be considered as a model of such structures as semi-submersible rigs, TLPs (tension-leg platforms) or column-supported VLFSs (very large floating structures).

2. Experiment

Water-tank experiments were conducted using an array of two rows of vertical truncated circular cylinders shown in Fig. 1 and Table 1. Four kinds of cylinder arrangement, that is, 2 × 9 cylinders, 2 × 7 cylinders, 2 × 5 cylinders, 2 × 3 cylinders were used. (Fig. 1 is the case of 2 × 9 cylinders.) The cylinders were fixed in regular head waves and the free-surface displacements at three points (Point A, Point B and Point C) in regular head waves were measured (see Fig. 2).

The experiments were conducted in two occasions. The first one (measurement at Point A) was conducted in the water tank (length: 45 m, width: 5 m) at the University of Tokyo and the second one (measurement at Points B and C) was conducted in the water tank (length: 100 m, width: 8 m) at the Yokohama National University.

The measured analog data on the free-surface displacements were converted to digital data with 10 ms time interval (e.g. 100 points in one period when the wave period was 1 s). The typical measurement time was 40 s (e.g. 40 oscillations when the wave period was 1 s), in which a steady-state part (typically 5–10 cycles) identified by visual judgment was Fourier analyzed numerically

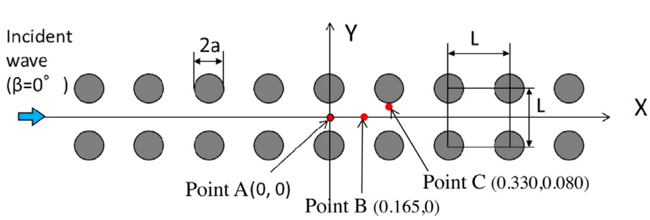

Fig. 2. Plan view of the experiment.

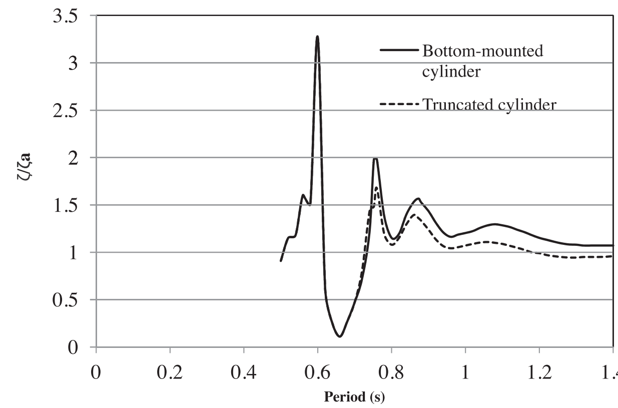

Fig. 3. Comparisons of first-order free-surface displacements obtained by the theory only applicable to bottom-mounted vertical cylinders with those obtained by a theory applicable to truncated cylinders.

and the first-order component (oscillating with the period of the incident waves) and the second-order component (oscillating with half the period of the incident waves) were extracted.

3. Theory

The theory used in the present calculations is not a new one but that presented by Sanada et al. [2], which extended the theory developed by Linton and Evans [9] for the first-order hydrodynamic interaction analysis among bottom-mounted vertical cylinders to the analysis of second-order wave diffraction. In the theory, exploiting the fact that the cylinders extend down to the sea bottom, the velocity potential of the bounded wave component of the second order around kth cylinder Real$(\phi_{2L}^{k}(r_k, \theta_k, z)e^{-2i\omega t})$, where ω stands for the radial frequency of the regular incident waves, is assumed to be expressed as follows:

$$\phi_{2L}^{k}(r_k, \theta_k, z) = f(r_k, \theta_k) \cdot \frac{\cosh 2k_1(z+h)}{\cosh 2k_1 h} \tag{1}$$

where (r_k, θ_k, z) is the local cylindrical coordinate system fixed to the kth cylinder and k_1 represents the wave number of the first-order wave component in finite water depth h, which satisfies the following dispersion relation:

$$\frac{\omega^2 h}{g} = k_1 h \cdot \tanh k_1 h \tag{2}$$

g represents the gravitational acceleration. (The time independent component of the second-order velocity potentials is not dealt with in the present study, since it does not contribute to the free-surface dynamic displacement.)

$f(r_k, \theta_k)$ in Equation (1) is determined in such a way that ϕ_{2L}^{k} satisfies the second-order inhomogeneous free-surface condition in an explicit form in terms of the first-order velocity potential.

The total velocity potential around the kth cylinder is expressed as the sum of ϕ_{2L}^{k} and the free wave components, which satisfy the homogeneous free-surface condition, including the free waves coming from the other cylinders. The undetermined coefficients involved in the expression of the free waves are determined so that the total velocity potential satisfies the non-penetrating condition on the wetted surface of the kth cylinder.

This solution is an approximate one in that ϕ_{2L}^{k} does not exactly satisfy the non-penetration conditions on body surfaces.

This theory (Theory A) is for bottom-mounted vertical cylinders and therefore, in a strict sense, cannot be used for the present cases, in which the cylinders are truncated. In Fig. 3, comparisons of first-order free-surface displacements obtained by the theory with those obtained by a theory applicable to truncated cylinders