Applied Ocean Research 47 (2014) 255–269

Contents lists available at ScienceDirect

Applied Ocean Research

journal homepage: www.elsevier.com/locate/apor

Constrained optimal stochastic control of non-linear wave energy point absorbers

M.T. Sichani[a,*], J.B. Chen[b,c], M.M. Kramer[a], S.R.K. Nielsen[a]

[a] Department of Civil Engineering, Aalborg University, 9000 Aalborg, Denmark
[b] School of Civil Engineering, Tongji University, Shanghai 200092, PR China
[c] State Key Laboratory of Disaster Reduction in Civil Engineering, Tongji University, Shanghai 200092, PR China

ARTICLE INFO

Article history:
Received 25 June 2013
Received in revised form 25 April 2014
Accepted 13 June 2014
Available online 11 July 2014

Keywords:
Wave energy point converter
Nonlinear buoyancy force
Constrained optimal stochastic control
Irregular sea state

ABSTRACT

The paper deals with the stochastic optimal control of a wave energy point absorber with strong nonlinear buoyancy forces using the reactive force from the electric generator on the absorber as control force. The considered point absorber has only one degree of freedom, heave motion, which is used to extract energy. Constrains are enforced on the control force to prevent large structural stresses in the floater at specific hot spots with the risk of inducing fatigue damage, or because the demanded control force cannot be supplied by the actuator system due to saturation. Further, constraints are enforced on the motion of the floater to prevent it from hitting the bottom of the sea or to make unacceptable jumps out of the water. The applied control law, which is of the feedback type with feedback from the displacement, velocity, and acceleration of the floater, contains two unprovided gain parameters, which are chosen so the mean (expected value) of the power outtake in the stationary state is optimized. In order to ensure accuracy of the results for each configuration of the controller Monte Carlo simulations have been carried out for various sea-states and the final results have been presented in the paper. The effect of nonlinear buoyancy force – in comparison to linear buoyancy force – and constraints of the controller on the power outtake of the device have been studied in details and supported by numerical simulations.

1. Introduction

A wave energy converter (WEC) may be defined as a dynamic system with one or more degrees of freedom with the intention to convert the energy in the waves into mechanical energy stored in the oscillating system. A point absorber is a WEC with a size that is small compared to the dominating wave length. The power outtake is basically the conversion of this mechanical energy into electricity. The absorbers of the WEC are typically equipped with an electric power generator via a hydraulic force system. The reaction force from the latter influences the motion of the point absorber. Within certain ranges the reaction forces can be specified at prescribed values. In so-called reactive control these forces are used to control the motion of the WEC in a way that a maximum mechanical energy is supplied to the absorbers. With a certain loss due to friction in the hydraulic force actuators the control forces are next transferred to the generators, where they are converted into electric energy.

Basically, the reactive control may be either of the open-loop (feed forward) or of the closed-loop (feedback) type. Open-loop control implies that the control effort is brought forward based on observation (measurement) of the wave elevation. Open-loop does not affect the dynamics of the system, i.e. angular eigenfrequencies and structural damping ratios are unchanged by the control. Closed loop control is entirely based on the observed motion of the absorbers. Typically, this involves the displacement, velocity and acceleration components, which easily can be measured by accelerometer or laser vibrometer measurements onboard the floating devices. A closed loop control always changes the dynamics properties of the system (inertia, damping or stiffness parameters), as specified by the poles and zeros of the frequency response functions relating the wave excitation forces to the displacement responses of the absorber system.

Latching, independently proposed in [1,2], is probably the simplest and definitely the most investigated control strategy. The control is based on the observation of the hydrodynamic force, for which reason latching control should be classified as a open-loop control strategy. Latching control requires that the hydrodynamic force can be predicted at least a semi-wave period ahead of the present time. In broad-banded irregular sea-states this prediction is related with uncertainty, which may affect the stability of the control. Normally, merely the sea surface elevation in the vicinity of the converter is observed. This makes observation of the wave excitation force components difficult, due to the non-causal dependence on this quantity on the sea-surface elevation [3]. Further, the power

---

* Corresponding author. Tel.: +45 9940 8570; fax: +45 9814 8552.
E-mail addresses: mts@civil.aau.dk (M.T. Sichani), chenjb@tongji.edu.cn (J.B. Chen), mmk@civil.aau.dk (M.M. Kramer), soren.nielsen@civil.aau.dk (S.R.K. Nielsen).

0141-1187/$ – see front matter © 2014 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.apor.2014.06.005

outtake from the control changes between finite time intervals with zero and non-zero power production, which may cause problems for the mechanical implementation of the method.

The simplest closed loop control law is achieved by a so-called derivative controller, where the reactive control force is specified to be proportional to and opposite directed to the velocity of the WEC. The controller has insignificant influence on the eigenfrequency of the absorber, for which reason the controller only becomes optimal for frequencies in the auto-spectrum of the wave excitation force in the vicinity of the undamped eigenfrequency of the absorber. By augmenting the controller with a force component proportional to either the displacement (proportional control) or the acceleration (acceleration control) a broader spectrum of frequencies can be absorbed. Proportional control will change the stiffness of the absorber, and acceleration control changes the mass. In both cases the eigenfrequency of the absorber can be changed to a certain extent. Finally, so-called integral control can be introduced for which the control force component appears as a convolution integral of the absorber velocity with respect to a given impulse response function. It turns out that integral control needs to be introduced, if perfect phase locking between the wave excitation force and the velocity of the absorber is attempted at all frequencies.

The idea of extracting energy from the waves is very old and many WEC devices have been proposed in the past [4,5]. This has initiated commercial WEC projects using devices such as different buoy concepts, Oscillating-Water-Column (OWC) plants, the Pelamis [7], overtopping WEC types like the Wave Dragon [8], point absorber approaches used for the Wavestar device [9], or the SEAREV multi-degree-of-freedom point absorber device [10]. Many control strategies have been indicated and reviewed in [2,11].

The non-linear stochastic control of a single wave energy point converter without constraints on displacement and control force was considered in [12]. The expected value (the mean value) of the harvested power was used as objective for optimal control. In case of a linear buoyancy force and linear wave mechanics it was shown in the paper that the optimal controller at a given time is a feedback controller with feedback from the present displacement and acceleration and a non-causal feedback from all future velocities of the absorber within the considered control horizon. In order to circumvent the indicated non-causality a causal control law applicable for nonlinear buoyancy forces was proposed by a slight modification of the optimal controller. The basic property of the devised control law is to enforce the wave excitation force into phase with the velocity of the absorber to insure a constant power supply. The controller contains a single undetermined gain factor, which has to be optimized to given irregular sea-state in accordance with the chosen stochastic optimality criteria. The devised controller was shown to be optimal under monochromatic wave excitation for a specific choice of the gain parameter.

Hansen and Kramer [13] considered the influence of constrains on the control force on the mean power outtake of a Wavestar point converter based on a PD reactive control law. It was concluded that the constrains significantly influence the mean power outtake, and change the values of the optimal gain factors of the PD controller. Constraints on the control force need to be taken in consideration in praxis in order to prevent large structural stresses in the floater at specific hot spots with the risk of inducing fatigue damage, or because the demanded control force cannot be supplied by the actuator system due to saturation problems.

As argued by [14] the aim of the control system is to optimize the generated electrical power rather than the harvested power. A positive power harvest indicates a power flow from the ocean to the generator, whereas a negative power implies a power flow in the opposite direction. Both power flows are related to inevitable power losses. Applying a PD controller they demonstrated that the generated electric power can be increased significantly by using the generated electric power rather than the harvested power at the optimization of the gain parameters of the control law.

Classical optimal control has indeed been applied to wave energy absorbers before, even with Pontryagins maximum principle involved for the case of saturation in the power take-off. However, in all cases known to the authors the canonical equations (the equations for the state and co-state vector) of the related two-point boundary value problem have been solved numerically. The inherent non-causality of the control law has been handled by a state predictor (Kalman- or Luenberger filter). The idea of [12] is to provide a closed-form analytical solution to the control of a single point absorber with non-linear buoyancy in terms of the basic hydrodynamic functions, which are obtained numerically by a BEM analysis. The optimal control law is a pure feed-back controller, with feed-back from the present displacement and acceleration, and all future velocities, which makes the controller non-causal. Hence, a prediction of future velocities, but not of surface elevations, is required for the optimal control. Next, a closely related causal controller is suggested, which of course is sub-optimal. However, it is demonstrated that the suggested causal controller is close to optimal, and superior to any PD feed-back controller.

In the present paper the problem considered in [12] is revisited. The idea here is to consider the practical limitations on the WEC. Here instead of the harvested power the generated electrical power will be the objective at the optimization of the control law. Further, the described control law is modified somewhat to take the indicated necessary constraints on the control force into consideration. Depending on the water depth the WEC may hit the sea-bottom at large motions with the risk to damage the floater at the impact, or it may make unacceptable jumps out of the water. To prevent these events constraints have been imposed on the allowable displacements of the WEC in the optimization of the mean of the generated electric power. These limitations are of considerable importance since the thresholds of the control force cause a sudden change in system's momentum hence introduces transients into the motion of the device. This indeed changes behavior of the system and cannot be neglected in analysis of the devices. A similar effect will be expected if the device reaches its ultimates on its range of motion.

The problem with saturation of the control force may be formulated as an outcrossing of a stochastic process from fixed boundaries. However, derivation of an analytical solution for power outtake of such a system requires an extensive study that is beyond scope of this work. In [12] the mean harvested power was evaluated based on covariance information for the response process, which was evaluated analytically. Due to the strong nonlinearities introduced by the buoyancy force at finite displacements and the introduced constraints this approach is no longer applicable. To circumvent this difficulty, a computational approach has been taken into account and the optimal gain values for the proposed controller are estimated using Monte Carlo simulations. The effect of including nonlinearity of buoyancy force in the model on the mean power outtake of the device is studied in details. Effect of threshold level of the control force has been extensively studied in the paper. Proper ways of treating thresholds in simulations have been addressed. Extensive discussion on the effect of various parameters of the controller on power extraction of the device are provided. Finally, conclusions are supported by large number of simulations for various cases.

2. Equation of motion of a WEC

2.1. Hydrodynamic forces

The motion of the WEC is described relative to the (x, y, z)-coordinate system shown in Fig. 1. The figure shows a cylindrical heave absorber in the static and dynamic deformed