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A probability-based superposition model of freak wave simulation☆

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ABSTRACT

The superposition model can generate freak waves in the specified location and time, thus becoming a common model for simulation of freak waves numerically and experimentally. However sometimes the superposition model does not meet the actual wave energy of random sea states and the simulation efficiency may not be high when the energy of focusing wave train is low. In this paper, the occurrence probability of freak waves generated using the superposition model is obtained based on which a method is developed to determine the number of wave components of the focusing wave train. The probability-based superposition model reduces energy proportion of the focusing wave train, and improves the simulation efficiency of freak waves based on the superposition model. Using the probability-based superposition model, a numerical wave tank (NWT) is created which solves the 2-D Navier–Stokes equations. Validity of the probability-based superposition model is proved by comparing the simulated freak waves with freak waves recorded both under the sea states of Japan and in the laboratory. When establishing the numerical wave tank, a new wave-absorbing method is developed to reduce the length of the absorbing region and to determine the absorbing weight.

1. Introduction

Freak waves are giant waves in the ocean which appear in seas across the world and bring great destruction to ships and marine structures [1]. Usually freak waves are defined as waves whose wave heights are twice higher than the significant wave height [2]. A large collection of freak wave observations show the main features of the freak wave phenomenon: appearance of large wave heights, often with holes, solitary pulses or a group of large amplitude waves, rare and short-lived, found in basins of arbitrary depth. Because the measured data of freak waves in situ are still not much, generation of freak waves in the laboratory or with numerical methods is important.

The numerical wave tanks are widely used because of their convenience, good repeatability and simulation in a realistic scale. It is a common method for numerical wave tanks to generate waves with a random phase approach using a realistic wave spectrum (e.g., P-M spectrum) based on the Longuet-Higgins model. However the freak wave would happen once in nearly 3000 waves according to the Rayleigh wave height distribution. Therefore, it is unrealistic to simulate freak waves with a fully random phase approach, besides some research showed the actual occurrence probability of freak waves was more than 1/3000 [3]. Alternatively, the superposition model is often used to generate freak waves. Li et al. [4] generated freak waves in the laboratory with the superposition model. Cui et al. [5] generated freak waves in the laboratory and studied the nonlinear effect of freak waves on the speed. Zhao et al. [6] simulated freak waves numerically using the superposition model. Cui et al. [5,7] simulated freak waves using the superposition method and studied the time-frequency energy of freak waves under different water depth. Fochesato et al. [8] simulated 3-D freak waves by superposing wave trains in different directions. Sun et al. [9] simulated freak waves using the superposition method and studied the evolution of freak waves. The superposition model divides waves into two trains: the focusing train and the random train. The energy of a realistic wave spectrum is proportional assigned to each wave train, which increases the occurrence probability of freak waves. Nevertheless, the superposition model has raised some questions that deserve further investigation. The energy portion of the focusing wave train is often relatively high which is unrealistic and would lower the significant wave height of the total wave train. With lower significant wave height, the simulated wave spectrum and the target wave spectrum often do not agree very well. Liu et al. [10] improved the superposition model by applying focusing wave trains with specific phases that superposed positively at given time and location, and a good agreement was get between the simulated wave spectrum and the target wave spectrum. However, the proportion of the focusing wave train was still big. There is still a problem for simulating freak waves in a numerical wave tank. To obtain a correct simulated wave spectrum, the simulation

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time needs to be long enough. Traditional wave-absorbing method may require a long absorbing region which would take much time during the freak wave simulation. Besides the spatial weight of absorbing region is hard to decide, and may need repeated trial.

This study is motivated by the desire to address the above-mentioned issues. In this paper, the occurrence probability of freak waves generated using the superposition model is deduced and a probability-based method of determining the number of wave components is developed to decrease the energy proportion of the focusing wave train. A conserved wave-absorbing method is introduced in the numerical wave tank. The absorbing weight is calculated by solving the mass conservation equation at any time step, which avoids repeated trial and reduces the absorbing area. Freak waves measured both under the sea states of Japan and in the laboratory are simulated in the numerical wave tank with the probability-based model, and the results show a good agreement.

2. The probability-based superposition method

2.1. Occurrence probability of freak waves

Before any further research, there must be a mathematical definition of freak wave. In this paper, we adopt the most popular amplitude criterion: its height should exceed the significant wave height in 2 times. The widely-used superposition model divides the wave elevation into the random wave train and the focusing wave train with different energy proportion as:

$$\eta = \sum_{i=1}^{N_1} a_{1i}\cos(k_{1i}x - \omega_{1i}t + \theta_{1i}) + \sum_{i=1}^{N_2} a_{2i}\cos[k_{2i}(x - x_c) - \omega_{2i}(t - t_c)] \tag{1}$$

where the first sum is the random wave train and the second sum is the focusing wave train. The subscript 1 and 2 denote the random wave train and the focusing wave train, a_{1i} and a_{2i} are the amplitudes of the ith wave components, k_i, ω_i and θ_i denote the wave number, frequency and phase of the ith wave component, constant x_c and t_c are the focusing location and time, N_1 and N_2 are the number of the wave components which are usually taken as the same value. The amplitudes of wave components are often decided by realistic wave spectrum like P-M spectrum whose energy is divided into two wave trains proportionally as:

$$\begin{cases} a_{1i} = \sqrt{2p_1 S(\omega_{1i})\Delta\omega_1} \\ a_{2i} = \sqrt{2p_2 S(\omega_{2i})\Delta\omega_2} \end{cases} \tag{2}$$

The proportion p_1 and p_2 would influent the wave heights of freak waves and their occurrence probabilities. If p_2 is small enough to be ignored, the whole wave train tends to be a random train, and thus the occurrence probability of freak waves is 1/3000 according to the Rayleigh-distributed wave height, which is smaller than their actual occurrence probability. To analyze the influence of p_2 quantitatively, take the P-M spectrum $S(\omega) = (A/\omega^5)e^{-B/\omega^4}$ for instance, where A and B are constants. When $x = x_c$ and $t = t_c$, the maximum surface elevation of the focusing wave train is obtained as:

$$\eta_{2\max} = \sum_{i=1}^{N} \sqrt{2p_2 \frac{A}{{\omega_{2i}}^5} e^{-B/\omega_{2i}^4}\Delta\omega_2} \tag{3}$$

If the number of wave components is big enough, the right side of formula (3) could be written in integral form, that is, $\eta_{\max} = (1/\sqrt{\Delta\omega_2})\int_0^\infty \sqrt{(2p_2A/\omega^5)e^{-B/\omega^4}}\,d\omega$. According to the definition of Gamma Function, the integration is written as:

$$\eta_{\max} = \frac{\sqrt{2p_2A}}{4\sqrt{\Delta\omega_2}}\left(\frac{B}{2}\right)^{-3/8}\Gamma\left(\frac{3}{8}\right) \approx 1.0868\sqrt{\frac{p_2A}{\Delta\omega_2}}B^{-3/8} \tag{4}$$

Take γ as the ratio of the minimum surface displacement and the maximum surface displacement, that is, $\gamma = \eta_{\min}/\eta_{\max}$, and the wave height of the focusing wave train is:

$$H_2 = (1-\gamma)\eta_{\max} \approx 1.0868(1-\gamma)\sqrt{\frac{p_2A}{\Delta\omega_2}}B^{-3/8} \tag{5}$$

The actual wave height is the supposition of the wave heights of the random wave train and the focusing wave train. The supposition is not a simple summation and Fig. 1 gives three situations of the supposition. The total wave height after supposition may be bigger (Fig. 1a), smaller (Fig. 1b) or basically unchanged (Fig. 1c), depending on the phase difference of the two wave trains.

According to the supposition situations above, the total wave height can be written as:

$$H = \alpha H_1 + H_2 \tag{6}$$

where H_1 is the wave height of the random wave train, and $\alpha = \cos\,\theta$. As θ is uniformly distributed in the interval $[0, 2\pi]$, the probability density function of α is $f_\alpha(\alpha) = 1/(\pi\sqrt{1-\alpha^2})$. Typically, the wind wave spectrum is assumed to be narrow, thus probability density function of the H_1 is defined by the Rayleigh distribution, that is, $f_{H1}(H) = (H/4m_0)e^{-H^2/8m_0}$, where $m_0 = p_1A/4B$ is the spectrum variance. Submitting the expression of $f_\alpha(\alpha)$ and $f_{H1}(H)$ into (6), the probability density function of the total wave height is the Gauss distribution as:

$$f_H(H) = \frac{1}{\sqrt{8\pi m_0}}e^{-(H-H_2)^2/8m_0} \tag{7}$$

And the cumulative probability function is given as:

$$F_H(H) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{H-H_2}{\sqrt{8m_0}}\right)\right] \tag{8}$$

where erf is the error function. Fig. 2 analyzes the wave heights of 1500 freak waves generated by numerical simulation, and compares the statistics with formulas (7) and (8), where $H_2 = 2.28$ cm, $p_1 = 0.8$ and $p_2 = 0.2$. The numerical and theoretical results of the probability density function and the cumulative probability function have a good agreement with each other.

According to Rayleigh distribution, the significant wave height is $H_s = 4\sqrt{m_0}$, with (5) and (8), the occurrence probability of freak waves is written as:

$$P_{\text{freak}} = \frac{1}{2}\left\{1 - \mathrm{erf}\left[2.8284 - 0.7685(1-\gamma)\sqrt{\frac{p_2}{p_1\Delta\omega_2}}B^{1/8}\right]\right\} \tag{9}$$

where B is decided by the frequency ω_m which is corresponding to the peak of wave spectrum. By getting the extreme value of the wave spectrum, B is given as $B = 1.25\omega_m^4$, thus (9) is rewritten as:

$$P_{\text{freak}} = \frac{1}{2}\left\{1 - \mathrm{erf}\left[2.8284 - 0.7902(1-\gamma)\sqrt{\frac{p_2\omega_m}{p_1\Delta\omega_2}}\right]\right\} \tag{10}$$

The ratio γ is also decided by ω_m, but it is difficult to give explicit expressions. The dependence is shown in Fig. 3 and when ω_m is small (<2 rad/s), the curve is close to the parabola, and the change of γ is in $[-0.43, -0.38]$, which is not very much.