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Reflection of oblique ocean water waves by a vertical porous structure placed on a multi-step impermeable bottom

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ABSTRACT

Based on linear water wave theory, wave reflection by a vertical porous structure placed on an elevated impermeable seabed, assumed to consist of a number of horizontal steps, is considered. The reflection phenomenon is investigated for two forms of the bottom topography: first a 2-step one and then a p-step one. An oblique incident wave propagates through the porous structure and gets reflected by the steps and a vertical solid wall which supports the structure at one end. Boundary value problems are set up in the two different media, the first medium being water and the second medium being the porous structure consisting of p vertical regions – one above each step. By using the matching conditions along the vertical boundaries, a system of linear equations is deduced. Reflection coefficient is obtained by solving this system of equations. The behavior of the reflection coefficient due to different relevant parameters is studied: the effect of various parameters, such as depth, porosity, number of evanescent modes, structure width and angle of incidence is studied graphically for both cases. It is observed that when the porous structure is considered above a 2-step bottom, the number of evanescent modes, porosity and the angle of incidence do not affect the reflection coefficient for relatively long waves. Lower values of friction factor results in oscillation in the reflection coefficient which vanishes with an increase in the values of friction factor. Up to a certain range of the angles of incidence, reflection coefficient is independent of the values of porosity. When a p-step bottom is considered, certain observations remain the same except that the behavior of reflection coefficient against the angle of incidence is different. In the end, both cases are compared by considering the same set of parameters. Justification of our model is presented by matching it with an available one.

1. Introduction

The concept of porous media is used in many areas of applied science and engineering: filtration, mechanics (acoustics, geomechanics, soil mechanics, rock mechanics [1]), engineering (petroleum engineering, bioremediation [2], construction engineering), geosciences (hydrogeology, petroleum geology [3], geophysics), biology, biophysics [4], material science, etc. These works amply justify that fluid flow through porous media is a subject of immense common interest and has, hence, emerged as a separate field of study. In coastal areas, porous structures are widely used as breakwaters to protect harbors, inlets and beaches from wave action, and as dissipating sea-walls to attenuate the wave energy in harbors. Because of this enormous significance of interaction of porous structures with ocean waves, we are motivated to investigate some specific cases of reflection by a porous structure placed on a step-like topography.

Many aspects of interaction between waves and porous media have been studied extensively. Theoretical solutions for reflection and transmission coefficients for certain types of porous structures have been analyzed previously by a number of researchers. The most widely used model of wave-induced flow in porous medium is the one developed by Sollitt and Cross [5]. According to their approach, dissipation of wave energy inside a porous medium is taken into account through a linearized friction term f which is evaluated by an iterative procedure. Madsen [6] derived a simple solution for reflection and transmission from a rectangular porous structure under normal incidence of long waves based on the linearized form of the governing equations and a linearized form of the flow resistance formula. Madsen [7] obtained a theoretical solution for the reflection of linear shallow-water waves from a vertical porous wave absorber placed on a horizontal bottom. The friction term describing the energy loss inside the absorber was linearized and thereafter, by using Lorentz principle of equivalent work, the reflection coefficient was determined as a function of parameters describing the incoming waves and the absorber characteristics.

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Kirby and Dalrymple [8] investigated the diffraction of obliquely incident surface waves by an asymmetric trench in which they developed a numerical solution by matching the particular solution for each subregion of constant depth along the vertical boundaries. An approximate solution based on plane-wave modes was derived and compared with the numerical solution. Sulisz [9] formulated a theory to predict wave reflection and transmission at an infinite rubble-mound breakwater under normal wave incidence. Dalrymple et al. [10] adopted the approach of Sollitt and Cross [5] to analyze the reflection and transmission of oblique incident waves from infinitely long porous structures. Losada et al. [11] extended this study to the case of an infinitely long, homogeneous, vertical structure capped with an impervious element under oblique incident wave. Mallayachari and Sundar [12] took into account the effects of an uneven seabed. The variation of the reflection coefficient with respect to the porosity of the wall, its friction factor and the relative wall width was studied. Zhu [13] used wave induced refraction-diffraction equations for surface waves in the region occupied by a porous structure. He utilized the orthogonality of the depth-dependent functions. Liu et al. [14] examined the hydrodynamic performance of a modified two-layer horizontal-plate breakwater consisting of an upper submerged horizontal porous plate and a lower submerged horizontal solid plate. Liu and Li [15] extended this work by considering a double curtain-wall breakwater whose seaward wall was perforated and shoreward wall impermeable. Cho et al. [16] further extended the idea of Liu et al. [14] by considering the lower submerged horizontal plate as a porous one instead of an impermeable one. Das and Bora [17] investigated wave reflection by a rectangular porous structure placed on an elevated bottom and supported by a vertical wall. The variation of reflection with respect to the number of modes, porosity, structure width, etc, was studied. Das and Bora [18] also studied wave damping by a vertical rectangular porous structure placed near and away from a rigid vertical wall. They computed the reflection coefficients for various depths, structure width and porosity.

The objective of this work is to solve a water wave scattering problem due to the presence of a vertical porous structure, placed on a 2-step or a p-step horizontal bottom and supported by a rigid vertical wall at one end, and to study the reflection characteristics. The effect of various parameters, such as number of evanescent modes, porosity, friction factor, structure width, angle of incidence on the reflection coefficient is investigated and the results are presented graphically. To the best of the knowledge of the authors, no one has solved the problem of wave propagation through a porous structure that is placed on a step-like impermeable bottom.

2. Mathematical formulation

Though the present problem is a specific one, it is fairly important to discuss some general features and equations which usually arise in wave propagation in porous medium. Therefore, in the following subsections a brief description on the specific porous structure under consideration is presented along with these equations ahead of the formulation of the present problem.

2.1. Porosity and porous structure

A porous medium mostly consists of pores through which fluid (be it gas or liquid such as water) can pass. The skeleton part of the medium is mainly solid (but foam is considered to be porous though). A porous medium may be an aggregate of large number of particles like sand, gravels or a solid containing many capillaries such as porous rock. Many natural substances like sponge, soil, biological materials (bone, lungs, etc.) are few examples of porous material. There are many man-made porous materials such

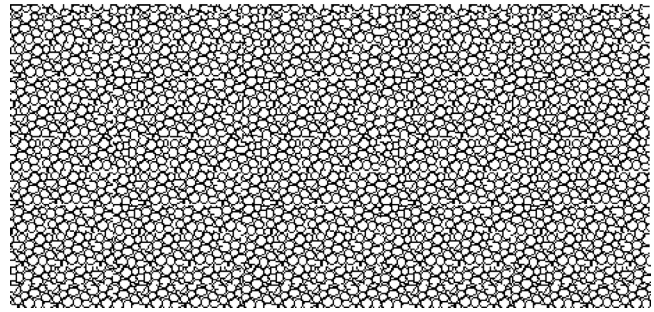

Fig. 1. Front view of the structural model.

as cement, ceramics, etc. Depending upon the pore size, porous structures can be categorized into three types, namely, microporous (smaller than 2 nanometers), mesoporous (between 2 and 50 nanometers) and macroporous (larger than 50 nanometers). Metallic foams are good examples of porous materials with higher porosity (ranging from 0.6 to 0.95). The construction of these types of materials can be performed through many means, the main one being the "lost-foam casting". Porous structures, with high porosity but with considerable stability, to be used as breakwater in ocean and coastal engineering can be constructed from low melting metals and alloys such as copper, aluminum, lead, tin, zinc, etc. The porous structure under consideration in this manuscript is taken as a such type of structure. Fig. 1 presents a rough visual representation of the present structural model.

2.2. General theory for flow inside porous medium

Small amplitude wave motion is considered within an undeformable porous medium. It is assumed that the porous structure is homogeneous and isotropic. The fluid motion follows the continuity equation and the equation of motion in terms of the seepage fluid velocity $\boldsymbol{U}$ and dynamic pressure P, which are given by

$$\nabla \cdot \boldsymbol{U} = 0, \tag{2.1a}$$

$$S\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\nabla P}{\rho} + f\omega \boldsymbol{U} = 0, \tag{2.1b}$$

where ρ is the density of the fluid, f is the linearized friction factor and ω is the angular frequency of the incident wave.

The inertial coefficient S is defined by

$$S = 1 + \frac{C_M(1-\gamma)}{\gamma}, \tag{2.2}$$

where C_M is the added mass coefficient and γ is the porosity of the porous structure. The physical significance and derivation of Eqs. (2.1a,b) and (2.2) are described in Appendix A.

A pore velocity potential $\Phi(x, y, z, t)$ is introduced to describe the wave-induced fluid motion in the porous medium:

$$\boldsymbol{U} = \nabla \Phi. \tag{2.3}$$

Integration of equation of motion (2.1b) leads to Bernoulli's equation

$$S\frac{\partial \Phi}{\partial t} + \frac{P}{\rho} + f\omega\Phi = 0. \tag{2.4}$$

The solution is assumed to be harmonic in time. So the fluid velocity, dynamic pressure and velocity potential can, respectively, be written as

$$\boldsymbol{U} = \boldsymbol{u}(x, y, z)\exp(-\mathrm{i}\omega t), \quad P = p(x, y, z)\exp(-\mathrm{i}\omega t),$$
$$\Phi = \phi(x, y, z)\exp(-\mathrm{i}\omega t), \tag{2.5}$$