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Fully nonlinear analysis of near-trapping phenomenon around an array of cylinders

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ABSTRACT

The wave diffraction around an array of fixed vertical circular cylinders is simulated in a numerical wave tank by using a fully nonlinear model in the time domain. The emphasis of the paper lies in the insightful investigation of the nonlinear properties of the near-trapping phenomenon associated with the multiple cylinders. The numerical model is validated by analytical solutions as well as experimental data for waves propagating past two and four vertical cylinders in certain arrangements. An array of four identical circular cylinders at the corners of a square with an incident wave along the diagonal of the square is the main focus here for investigating the near-trapping phenomenon. When near-trapping occurs, the present study shows that an extremely high wave elevation near the cylinders can be observed. At the same time, the hydrodynamic forces on different cylinders are found to be either in phase or out of phase, leading to some characteristic force patterns acting on the whole structure. Due to the nature of the numerical model adopted, nonlinearity at different orders can be captured using a harmonic analysis. In addition to first- and second-order near-trapping, the third-order (triple-frequency) nonlinear component is presented for the first time. For the configuration selected, it is found that at one specific incident wave frequency and direction one trapped mode is excited by second-order effects, while a different trapped mode (having similar symmetries) is excited by the third harmonic of the incident wave frequency.

1. Introduction

It has been several decades since researchers started to work on the problem of wave–structure interactions. The hydrodynamic properties of column-supported offshore structures have attracted much attention with growing practical experience in the offshore industry of column-based platforms such as tension leg platforms and semi-submersibles operating well in deep-water fields. In addition, one particular interest among other issues is the high wave elevation and significantly large wave force caused by the wave diffraction and re-diffraction due to the presence of multiple cylinders. Damage to the lowest deck of those multi-column platforms has been reported, as for example discussed in Swan et al. [1], and such cases could be due to underestimation of the prevailing wave climate, and/or the unreliable prediction of the maximum wave elevation and upwelling during the design of the platforms. It is the latter of these that concerns us here. It is linked to the phenomenon known as 'near-trapping', which has been observed for multi-column arrays in laboratory experiments, for example by Ohl et al. [2] and Kashiwagi and Ohwatari [3]. For some arrangements of an array of cylinders, for example with certain symmetries, near-trapping occurs at certain frequencies, known as near-trapping frequencies, at which only a small amount of scattered wave energy is radiated outwards to the far field: the wave is trapped within the local vicinity of the cylinders, forming a near standing wave with much larger amplitude compared with that at other frequencies. In the design of such multi-column platforms, therefore, it is crucial to develop a reliable tool to predict and describe the near-trapping phenomenon.

For an array of bottom-mounted vertical circular cylinders, pioneering work was done by Spring and Monkmeyer [4], who in 1974 derived the linear analytical solution for the scattered potential around circular cylinders. McIver and Evans [5] presented an approximation for estimating wave forces on a group of vertical cylinders with large distances between each one, and Eatock Taylor and Hung [6] investigated the mean wave drift forces on multi-column structures. Meanwhile, an accurate algebraic method was developed by Kagemoto and Yue [7] to predict the hydrodynamic properties of a system of multiple three-dimensional bodies in water waves. Based on the method in Spring and Monkmeyer [4], Linton and Evans [8] further derived a considerably simplified formula for the prediction of first-order and mean forces on multiple cylinders as well as for the calculation of the free surface profile.

Trapped modes were first identified in an open channel by Ursell [9] in 1951. Callan et al. [10] proved the existence of trapped modes in two-dimensional waveguides using Ursell's method. Subsequently,

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Evans et al. [11] theoretically showed the existence of trapped modes for all symmetric cylindrical cross-sections. Thereafter, the theory of Linton and Evans [8] was used by Evans and Porter [12] to study the first-order near-trapping for waves around an array of bottom-mounted vertical cylinders, while Maniar and Newman [13] observed similar near-resonant modes between adjacent cylinders in a long finite array at critical wave numbers. Consideration has also been given to the effects of near-trapping by such configurations in focused wave groups, e.g. Grice et al. [14]. Malenica et al. [15] extended the investigation of near-trapping for an array of equally spaced, identical circular cylinders to second-order in wave steepness. By studying the second-order diffraction of monochromatic waves via a semi-analytical approach, they solved for the waves due to the second-order potential, and suggested that there also exists a near-trapping phenomenon for the second-order wave around an array of cylinders. This second-order near-trapping occurs for an incident wave at half the corresponding first-order near-trapping frequency. Wang and Wu [16] undertook a second-order analysis of near-trapping of such an array in the time domain. Linear near-trapping by truncated cylinders has been considered by Siddorn and Eatock Taylor [17]. For more complex geometries, such as multi-column gravity platforms, semisubmersibles or tension leg platforms, and other trapping structures, numerical diffraction codes have been employed by several investigators.

For the problem of higher-order nonlinear wave diffraction, Malenica and Molin [18] formulated the theory for third-harmonic diffraction by a vertical cylinder based on the perturbation procedure. Meanwhile, Faltinsen et al. [19] proposed an approximate theory (now known as the FNV approximation), by assuming that the wave amplitude and the cylinder radius are of the same order. In order to consider fully nonlinear effects at different orders in the case of steep waves, a fully nonlinear time domain simulation may be the most appropriate method. This can consider all the nonlinearity of the problem without any approximations such as Taylor series expansion or perturbation procedure. So far, various fully nonlinear numerical models have been developed by different research groups. Ferrant [20] developed a fully nonlinear wave tank to study the diffraction problem and for a single vertical cylinder obtained the higher-order harmonics up to seventh order. Subsequently, Huseby and Grue [21] performed a large number of tests in a long wave tank and compared the higher-harmonic wave forces on a vertical cylinder against the nonlinear computations of Ferrant [20]. Good agreement, even at higher harmonics, was shown from the comparisons of the measurements and fully nonlinear computations. Ma et al. [22,23] studied the fully nonlinear wave diffraction around a pair of fixed cylinders in a numerical wave tank based on the finite element model (FEM). Applications of this approach to simulate wave interaction with a moving cylinder include Wu and Hu [24] and Wang et al. [25]. Thereafter Koo and Kim [26], Bai and Eatock Taylor [27,28] and Zhou et al. [29] used the boundary element method to simulate the fully nonlinear wave interaction with structures in 2D and 3D, respectively.

An investigation of near-trapping in a long array of cylinders was undertaken by Wang and Wu [30] using a fully nonlinear method based on a finite element model, and the time histories of force and wave run-up were provided for different situations. However, to the best of our knowledge, a systematic investigation of the nonlinear features associated with the near-trapping phenomenon has not been published. This paper attempts to shed light on this topic by employing the fully nonlinear time domain numerical model developed by Bai and Eatock Taylor [27,28] to investigate wave diffraction around an array of bottom-mounted cylinders. The adopted numerical model is validated by considering the cases of a pair of bottom-mounted vertical circular cylinders and an array of four identical circular cylinders situated at the corners of a square. Numerical results for the wave elevation, run-up and forces are presented, which are found to agree well with the experimental data. The near-trapping phenomenon in

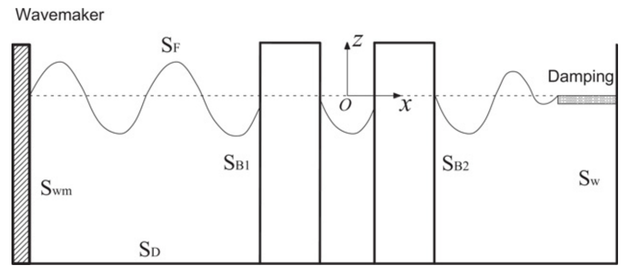

Fig. 1. Sketch of the numerical wave tank.

an array of cylinders is then studied, and results including the mode shape of the free surface elevation at near-trapping, and the pattern of forces on the cylinders, are obtained. In addition, a harmonic analysis of the time history of the fully nonlinear results is performed using the Fast Fourier Transform (FFT), from which the importance of higher-order effects can be identified, especially in the case of the second-order near-trapping. In this paper, unless otherwise specified, the terms first-order, second-order and third-order refer to the single frequency, double frequency and triple frequency components, respectively, of the fully nonlinear results obtained by using an FFT analysis. Thus first harmonic terms resulting from the third order interaction of first and second order terms are not considered to affect the overall conclusions drawn here about nonlinear behavior.

2. Mathematical formulation

A numerical wave tank defined in Fig. 1 is adopted to simulate the aforementioned wave diffraction around an array of vertical circular cylinders and the associated near-trapping phenomenon. The schematic figure involves a wave maker at the left boundary of the tank, an array of bottom-mounted vertical circular cylinders in the middle of the tank and a damping layer placed on the water surface to avoid wave reflection from the far end of the wave tank.

Based on the assumption that the fluid is incompressible and inviscid, and the flow irrotational within the fluid domain, potential flow theory can be used to describe the wave diffraction problem, where a velocity potential $\phi(x, y, z, t)$ defined in a global coordinate system $Oxyz$ satisfies the Laplace equation,

$$\nabla^2\phi = 0. \tag{1}$$

On the free water surface S_F, the kinematic and dynamic wave conditions in the Lagrangian description are

$$\frac{\mathrm{D}\mathbf{X}}{\mathrm{D}t} = \nabla\phi, \tag{2}$$

$$\frac{\mathrm{D}\phi}{\mathrm{D}t} = -gz + \frac{1}{2}\nabla\phi\cdot\nabla\phi, \tag{3}$$

where $\mathrm{D}/\mathrm{D}t$ is the material derivative, $\mathbf{X}$ denotes the position of water particles on the free water surface and g is the gravitational acceleration. As the bodies are bottom-mounted and the rigid boundaries should be impermeable, the kinematic condition on the body surfaces, bottom and sidewalls of the tank is

$$\frac{\partial\phi}{\partial n} = 0, \tag{4}$$

where $\mathbf{n}$ is the normal unit vector pointing out of the fluid domain. The initial condition is taken as

$$\phi = 0, z = 0 \quad \text{at } t = 0. \tag{5}$$

The higher-order boundary element method is employed to solve the mixed boundary value problem described above at each time step, where the surface over which the integral is performed is discretized